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ALGORITHMS FOR PIPE NETWORK ANALYSIS AND THEIR RELIABILITY

By

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DISCLAIMER

ABSTRACT

Algorithms for analyzing steady state flow conditions in pipe networks are developed for general applications. The algorithms are based on both loop equations expressed in terms of unknown flowrates and node equations expressed in terms of unknown grades. Five methods, which represent those in significant use today, are presented. An example pipe network is analyzed to illustrate the application of the various algorithms. The various assumptions required for the different methods are presented and the methods are compared within a common framework.

The reliabilities of these commonly employed algorithms for pipe network analysis are investigated by analyzing a large number of pipe networks using each of the algorithms. Numerous convergence and reliability problems are documented. It is shown that two methods based on loop equations have superior convergence characteristics. Methods based on node equations are less reliable and these methods are often unable to adequately handle low resistance lines. A discussion of convergence criteria and solution reliability is presented for the various algorithms. The results presented in this report will allow engineers to accurately carry out pipe network analyses.

Descriptors: water distribution, piping systems, networks, convergence, reliability

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INTRODUCTION

Steady state analysis of pressure and flow in piping systems is a problem of great importance in engineering. The basic hydraulics equations describing the phenomena are non-linear algebraic equations which can not be solved directly. These equations have been expressed in two principal fashions. They have been written in term of the unknown flowrates in the pipes herein referred to as loop equations. Alternately, they have been expressed in terms of unknown grades at junctions throughout the pipe system (node equations). Several algorithms have been proposed for solving these equations and these techniques are in wide use today.

In this report the principal algorithms are developed for applications to piping systems of general configuration and systems which contains pumps and other common hydraulic components. In most situations the algorithms have been previously presented with limitations on pipe system configurations and components which can be considered. An example system is analyzed using each of the algorithms to illustrate the required computational procedures. The efficiency and reliability of these algorithms are examined by solving a large number of actual pipe network problems with each algorithm and comparing the results.

There is a considerable amount of published material dealing with pipe network analysis, all of which can not be reviewed here. However, an attempt will be made to cite some principal contributions of historical interest. Hardy Cross authored the original and classic paper (1). In that article, which considered only closed loop networks with no pumps, a method for solving the loop equations based on adjusting flowrates to balance the energy equations is described. This method is very widely used today and is often referred to as the Hardy Cross method. Although it is not as widely used, Hardy Cross described a second method for solving the node equations by adjusting grades so that the continuity equations are balanced. A number of subsequent papers have appeared which further describe these methods or computer programs utilizing these methods (2, 3, 4, 5, 6).

Because the adjustments are computed independently from each other, convergence problems using the methods described by Hardy Cross were frequently noted and procedures developed to improve convergence. Martin and Peters (7) and Epp and Fowler (8) described a procedure to simultaneously compute the flow adjustments for closed loop systems. This procedure had much improved convergence characteristics and forms the basis for more general applications (5, 9).

Both the methods just described for solving the loop equations require an initial balanced set of flowrates and the convergence depends to a degree on how close this initial set of flowrate is to the correct solution. A third method, developed by this writer solves the entire set of hydraulics equations simultaneously after linearizing the energy equations. This procedure was first described for closed loop systems (10) and has subsequently been modified for more general applications (11, 12). A similar approach has been developed for the node equations where all the equations are linearized and solved simultaneously, and this method is described for closed loop systems by Shamir and Howard (13). Additional references to this method have been published (14, 15).

The five methods just described are developed in detail in the next section for general applications. No restrictions on network geometry are required and pumps may be included anywhere in the network. Certain special components such as check valves and pressure regulating valves are not considered since these components require special treatment.

Each of these methods for pipe network analysis discussed in this report require iterative computations whre the solution is improved until a specified convergence criterion is met. If a sufficiently stringent convergence criterion is met, the solutions normally will be essentially identical for all five methods. In some cases, however, it is not possible with certain algorithms to meet a specified convergence criterion regardless of the number of trials completed. In other cases a seemingly stringent convergence criterion may be met but the solution is still in considerable error. Convergence difficulties such as these have been previously noted and reported. In this report, results are presented of a detailed study that documents convergence problems and program efficiency. This study was carried out by comparing solutions obtained with the various algorithms using an extensive data base describing a variety of actual piping systems operating under widely varying conditions..

In the original paper by Hardy Cross he noted that "convergence was slow and not very satisfactory" when employing the single node adjustment method he developed (1). This was attributed to using initial grade estimates which were not very good. Of the two methods described by Hardy Cross, the method of balancing heads (single path adjustment method) became the most widely taught and used method. Convergence problems using this method were also recognized, however, and several suggestions were made for improving convergence. Investigators have advocated the use of an over-relaxation factor to multiply the flow adjustment factor (16, 17). Hoag and Weinberg suggested using a selective procedure for choosing paths as a means of accelerating convergence (18). It appears that these and other procedures suggested for improving convergence of the single path adjustment method will improve it only in certain situations and will not assure convergence.

Convergence problems are largely unreported for the improved methods developed for solving the loop equations. These are the simultaneous path adjustment and the linear methods which are included in the current study. However, additional convergence problems have been reported for methods based on the node equations since Hardy Cross first alluded to such problems in his original paper. Dillingham (4) stated that when the single node adjustment method is applied to a large network it may not converge or may converge very slowly. He described some procedures for improving convergence. Robinson and Rossum (6) who developed a computer program based on the single node adjustment method state that, "convergence is slow when a network contains short lengths of large diameter mains and cnvergence is not assured if there are dead end mains." They further state that "convergence may not occur if check valves are present." The simultaneous node adjustment method normally converges much more rapidly which lead Lemieux to state that convergence was assured (14). However, it appears that this assessment is optimistic and problems have been noted with this method.

Oscillations have been noted by Shamir and Howard (13) who also report that there is a possibility that a solution can not be obtained. Liu also stated "for poor initial input the method (simultaneous node adjustment) may diverge from the true solution or converge slowly" (15). Collins and Kennington presented some data which documented convergence problems for a large network using this method. (19).

The reliability of the algorithm employed for pipe network analysis if of great importance and the most single significant consideration. Failure to obtain a solution is a great inconvenience and the failure to recognize a poor solution may be even a greater problem because this may lead to poor design or management of water distribution systems. The purpose of this study is to document reliability problems which may occur using the various popular algorithms so that the frequency and severity of such problems will be known.

ANALYSIS

Pipe Network Geometry

Basic geometric considerations for a pipe network are summarized as follows. A pipe network is comprised of a number of pipe sections which are constant diameter sections which can contain pumps and fittings such as bends and valves. The end points of the pipe sections are nodes which are identified as either junction nodes or fixed grade nodes. A junction node is a point where two or more pipe sections joins and is also a point where flow can enter or leave the system. A fixed grade node is a point where a constant grade is maintained such as a connection to a storage tank or reservoir or to a constant pressure region. In addition primary loops can be identified in a pipe network and these include all closed pipe circuits within the network which have no additional closed pipe circuits within them. When junction nodes, fixed grade nodes, and primary loops are identified the following relationship holds:

$$p = j + \ell + f - 1 \tag{1}$$

where

p = number of pipes
j = number of junction nodes
ℓ = number of primary loops
f = number of fixed grade nodes

It turns out that this identity is directly related to the basic hydraulics equations which describe steady state flow in the pipe network.

Basic Equations

Pipe network equations for steady state analysis have been commonly expressed in two ways. Equations which express mass continuity and energy conservation in terms of the discharge in each pipe section have been referred to as loop equations and this terminology will be followed here. A second formulation which expresses mass continuity in terms of grades at junction nodes produces a set of equations referred to as node equations.

<u>Loop Equations</u> - Eq. 1 which defines the relationship between the number of pipes, primary loops, junction nodes and fixed grade nodes offers a basis for formulating a set of hydraulic equations to describe a pipe system. In terms of the unknown discharge in each pipe, a number of mass continuity and energy conservation equations can be written equaling the number of pipes in the system. For each junction node a continuity relationship equating the flow into the junction (Q_{in}) to the flow out (Q_{out}) is written as:

$$\Sigma Q_{in} - \Sigma Q_{out} = Q_e \qquad \text{(j equations)} \qquad (2)$$

Here Q_e represents the external inflow or demand at the junction node. For each primary loop the energy conservation equation can be written for pipe sections in the loop as follows:

$$\Sigma h_L = \Sigma E_p \qquad (\ell \text{ equations}) \qquad (3)$$

where

h_L = energy loss in each pipe (including minor loss)
E_p = energy put into the liquid by pumps

If there are no pumps in the loop then the energy equation states that the sum of the energy losses around the loop equals zero.

If there are f fixed grade nodes, f - 1 independent energy conservation equations can be written for paths of pipe sections between any two fixed grade nodes as follows:

$$\Delta E = \Sigma h_L - \Sigma E_p \qquad \text{(f-1 equations)} \qquad (4)$$

where ΔE is the difference in total grade between the two fixed grade nodes. Any connected path of pipes within the pipe system can be chosen between these nodes. When identifying these f - 1 energy equations care must be taken to avoid redundant paths. The best method to avoid this difficulty is to either choose all parallel paths starting at a common node (like A-B, A-C, A-D, etc) or to use a series arrangement where the previous ending node for a path is the starting node for the next path (like A-B, B-C, C-D, etc.). Either of these methods will results in f - 1 equations with no redundancy.

As an additional generalization Equ. 3 can be considered to be a special case of Equ. 4 where the difference in total grade (ΔE) is zero for a path which forms a closed loop. Thus, the energy conservation relationships for a pipe network are expressed by $\ell + f - 1$ energy equations of the form given by Equ. 4. The continuity and energy equations constitut a set of p simultaneous nonlinear algebraic equations referred to as loop equations which describe steady state flow conditions within a system of pipes. A steady state flow analysis based on the loop equations requires the solution of this set of equations for the flowrate in each line. To do this the terms in the energy equations must be expressed as functions of the flowrate which is done as follows.

The energy loss in a pipe (h_L) is the sum of the line loss (h_{LP}) and the minor loss (h_{LM}). The line loss expressed in terms of the flowrate is given by:

$$h_{LP} = K_P Q^n \tag{5}$$

where K_P is a pipe line constant which is a function of line length, diameter, and roughness, and n is an exponent. The values of K_P and n depend on the energy loss expression used for the analysis. Commonly used expression for this include the Darcy-Weisbach, Hazen Williams and Manning Equations.

The minor loss in a pipe section (h_{LM}) includes losses at fittings, valves, meters and other components which disturb the flow and is given as

$$h_{LM} = K_M Q^2 \tag{6}$$

where K_M is the minor loss constant which is a function of the sum of the minor loss coefficients for the fittings in the pipe section (ΣM)

and the pipe diameter and is given by

$$K_M = \Sigma M/2gA^2 \tag{7}$$

where A is the cross-sectional pipe area.

Pumps are described several ways. In some cases a constant power input is specified. In other cases a curve is fit to actual pump operating data. A variety of functions have been suggested for the curve chosen to fit the data and a common choice is a second order polynominal. For all the applications involving pumps the relationship between the pump energy, E_p, and discharge, Q, can be specified by a reasonably simple expression

$$E_p = P(Q) \tag{8}$$

Utilizing Equs. 5-8 the energy equations expressed in terms of the flowrate are

$$\Delta E = \Sigma(K_p Q^n + K_M Q^2) - P(Q) \tag{9}$$

The continuity equations (Equ. 2) and the energy equations (Equ. 9) form the set of p simultaneous equations in terms of unknown flowraes which are termed the loop equations. Since these are nonlinear algebraic relationships no direct solution is possible. Three algorithms for solving the loop equations are presented in this paper.

Node Equations - The analysis is carried out in terms of an unknown total grade, H, at each junction node in the piping system. The basic relationship used is the continuity relationship (Equ. 2)

$$\Sigma Q_{in} - \Sigma Q_{out} = Q_e$$

The flowrate in a pipe section connecting nodes labeled a and b is expressed in terms of the grade at junction node a, H_a, the grade at the other end of the pipe section, H_b, and the resistance of the pipe, K_{ab}.

This is

$$Q_{ab} = (H_a - H_b)/K_{ab} \quad ^{1/n} \tag{10}$$

This expression assumes that the pipe section contains no pumps and a head loss relationship is used of the form

$$h_L = K\,Q^n \tag{11}$$

where the term K is the loss coefficient for the pipe section and is a function of pipe parameters and flow conditions and depends on the head loss expression used and may include minor loss terms. The exponent n also depends on the head loss expression used.

Combining Equs. 2 and 10 gives:

$$\sum_{b=1}^{N} \pm \left| \frac{H_a - H_b}{K_{ab}} \right|^{\frac{1}{n}} = Q_e \qquad \text{(j equations)} \tag{12}$$

This expresses continuity at junction node a where N pipes connect in terms of the grade at a, H_a, and the grades at adjacent nodes, H_b. The sign of the term in the summation depends on whether the flow is into or out of junction node a. A total of j equations are written in this manner.

The basic set of equations can be expanded to incorporate pumps. For each pump junction nodes are identified at the suction and discharge sides of the pump as shown in Figure 1. Two additional equations can be written in terms of the two additional unknown grades at the suction and discharge sides of the pump and the adjacent grades.

Using the notation shown in Figure 1, one equation utilizes flow

Figue I Pump Notation for Node Equations

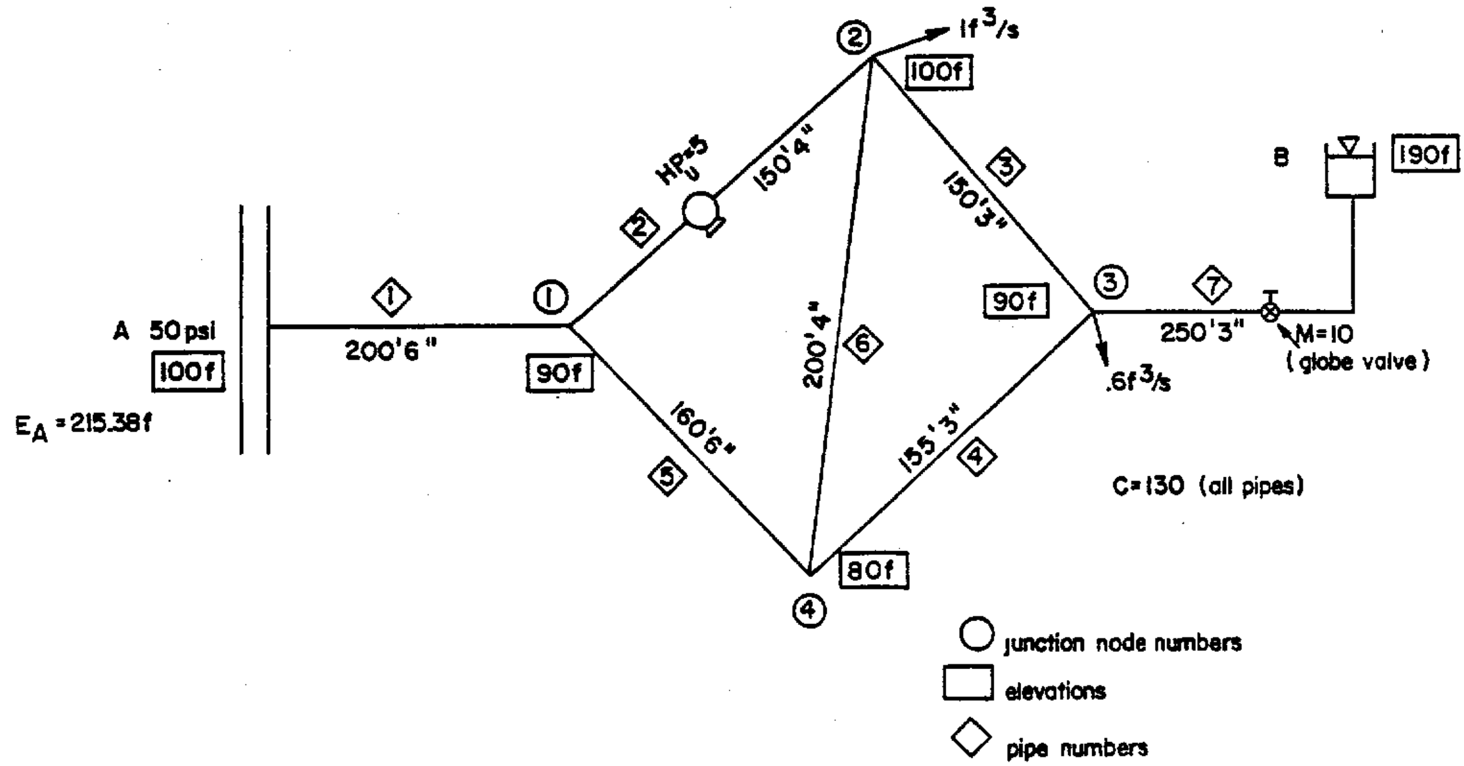

Figure 2 Example Pipe System (1f. = .3048m, 1 in. = 2.54cm, 1f^3/s = .028m^3/s, 1#/in^2 = 689.5 N/m^2, 1 Hp = .746 KW)

continuity in the suction and discharge lines to give

$$H_a - H_b = \frac{K_{ab}}{K_{cd}} (H_c - H_d) \tag{13}$$

A second equation relates the head change across the pump to the flow in either the discharge or suction line. For pumps operating at constant power this relation in terms of the discharge line flow is

$$H_c - H_b = P\left(\left| \frac{H_c - H_d}{K_{cd}} \right|^{\frac{1}{n}} \right) \tag{14}$$

Equations 12-14 represent the full set of pipe network node equations which are expressed in terms of the unknown grades at junction nodes and pump suction and discharge grades at all pumps in the pipe system. Like the loop equations, these are nonlinear algebraic equations and no direct solution is possible.

Algorithms for the Solution of the Loop Equations

Three methods for the solution of the loop equations have been developed and are in significant use today. Each of these use gradient methods to handle the non-linear flowrate (Q) terms in the energy equation (Equ. 9). The right side of Equ. 9 represents the grade difference across a pipe section carrying a flowrate Q. This is

$$f(Q) = K_P Q^n + K_M Q^2 - P(Q) \tag{15}$$

The function and its gradient evaluated at an approximate value of the flowrate, Q_i, are used in all the algorithms presented for solving the loop equations. The grade difference in a pipe section based on $Q=Q_i$ is:

$$f(Q_i) = H_i = K_P Q_i^2 - P(Q_i) \tag{16}$$

and the gradient evaluated at $Q = Q_i$ is:

$$f'(Q_i) = G_i = \left.\frac{\partial f}{\partial Q}\right|_{Q=Q_i} = nK_P Q_i^{\,n-1} + 2K_M Q_i - P'(Q_i) \tag{17}$$

The terms H_i and G_i as defined above are referenced in the following discussion of the algorithms for the loop equations and the example calculations which illustrate the application of these algorithms to pipe network problems.

Single Path Adjustment (P) Method. - This method of solution was described by Hardy Cross (1) and is the oldest and most widely used technique. The original method was, however, limited to closed loop systems and included only line losses. Herein the procedure is generalized. The method of solution is summarized as follows:

1. Determine an initial set of flowrates which satisfy continuity at each junction node.

2. Compute a flow adjustment factor for each path ($\ell + f - 1$) in the pipe system which tends to satisfy the energy equation written for that path. The application of this correction factor will not disturb the continuity balance.

3. Using improved solutions for each trial repeat step 2 until the average correction factor is within a specified limit.

The adjustment factor for a path is computed from equation 9 using a gradient method for solving a nonlinear equation for a single unknown. This method is based on the following approximation for the terms in equation 9 which are functions of the flowrate Q.

$$f(Q) - f(Q_i) + \left.\frac{\partial f}{\partial Q}\right|_{Q=Q_i} \Delta Q \tag{18}$$

where Q_i is an approximate value of the flowrate. Applying Equ. 18 to the line loss, minor loss and pump energy terms in Equ. 9 and solving for ΔQ gives:

$$\Delta Q = \frac{\Delta E - \Sigma H_i}{\Sigma G_i} \tag{19}$$

The Σ means that the contribution from each pipe in the path must be included. The terms ΔQ represents the flowrate adjustment factor which must be applied to each pipe in the path. The numerous represents the unbalance in the energy relationship due to the incorrect flowrates and the procedure is developed to reduce this to a negligible quantity. A trial with this method requires a flow adjustment to all paths in the pipe network (ℓ loop and f - 1 paths between fixed energy nodes).

Simultaneous Path Adjustment (SP) Method. - In order to improve the convergence characteristics a method of solution was devised which simultaneously adjusts the flowrate in each path of pipes representing an energy equation (2). This method can be summarized as follows.

1. Determine an initial set of flowrates which satisfy continuity at each junction mode.

2. Simultaneously compute a flow adjustment factor for each path which tends to satisfy the energy equations without disturbing the continuity balance.

3. Using the improved solutions repeat step 2 until the average flow adjustment factor is within a specified limit.

The simultaneous determination of the path flow adjustment factors requires the simùltaneous solution of $\ell + f - 1$ equations. Each equation accounts for the unbalance in the energy equation due to incorrect values of flowrate. The equation includes the contribution for a particular path as well as contributions from all other paths which have pipes common to both paths. Gradient techniques are used to formulate these equations. For path j, the head change required to balance the energy equation is expressed in terms of the flow change in path j (ΔQ_j) and the flow changes in adjacent paths (ΔQ_K) and is given as:

$$\Delta H = \frac{\partial H}{\partial Q}\Bigg|_{Q=Q_i} \Delta Q_j \quad + \quad \Sigma \frac{\partial H}{\partial Q}\Bigg|_{Q=Q_i} \Delta Q_k \tag{20}$$

This can be expressed in terms of H_i and G_i for the pipes in the path as follows

$$\Delta H = \Delta E - \Delta H_i = (\Delta G_i)\Delta Q_i + \Delta (G_i \Delta Q_K) \tag{21}$$

Here ΣH_i represents the algebraic sum of the head changes for all the pipes in path j, $(\Sigma G_i)\Delta Q_j$ represents the sum of all the gradients for the same pipes times the flow change for that path and $\Sigma(G_i \Delta Q_K)$ represents the sum of the gradients for pipes common to paths j and K times the flow change for path K. The last term is repeated for all paths with pipes common to path j.

In this manner a set of simultaneous linear equations are formed in terms of flow adjustment factors for each path representing an energy equation. These linear equations can be solved using standard procedures and the solution provides an improved set of balanced flowrates which can be used for another trial. Trials are repeated until a specified accuracy is attained.

Linear (L) Method. - This method is based on a simultaneous solution of the basic hydraulics equations for the pipe system and has been reported for closed loop systems (3) and general systems (4). Since the energy equations for the paths are non-linear, these equations are first linearized in terms of an approximate flowrate, Q_i, in each pipe. This is done by taking the derivative of the variables in Equ. 9 with respect to the flowrate and evaluating them at $Q = Q_i$ using the following approximation

When this relationship is applied to the energy equation (Equ. 9) the following linearized equation results:

$$\Sigma G_i Q = \Sigma(G_i Q_i - H_i) + \Delta E \tag{22}$$

The Σ refers to each pipe in the path. Equ. 23 is employed to formulate $\ell + f - 1$ energy equations which combine with the j continuity equations (equs. 2) to form a set of p simultaneous linear equations in terms of the flowrate in each pipe.

The technique used to solve the system equations follows. Based on an arbitrary initial value for the flow in each line the linearized equations are solved using routing matrix procedures for solving linear equations. This set of flowrates is used to linearize the equations and a second solution is obtained. The procedure is repeated until the change in flowrates obtained in successive trials is insignificant.

Algorithms for the Solution of Node Equations

Two methods for solving the node equations are widely used and these are described here.

Single Node Adjustment (N) Method - This method was also described in the paper written by Hardy Cross (1). However, the method has never been widely used as the single path adjustment method. It is, nevertheless, in significant use today. The method is summarized as follows:

1. Assume a reasonable grade for each junction node in the system. This assumed grade does not have to satisfy any conditions. However, the better the initial assumption the fewer the required trials.

2. Compute a grade adjustment factor for each junction node which tends to satisfy flow continuity.

3. Repeat step 2 until the average correction factor for grades is within a specified accuracy or some other specified convergence criterion is satisfied.

The grade adjustment factor is the change in grade at a particular node (ΔH) which will results in satisfying continuity (Equ. 2) considering the grades at the adjacent nodes as fixed. Again a gradient approximation is used to compute the required grade change. This is:

$$f(H) = f(H_i) + \frac{\partial f}{\partial H}\bigg|_{H=H_i} \Delta H \qquad (25)$$

It is convenient to express the grade correction factor in terms of Q_i which is the flowrate based on the values of the grades at adjacent nodes before adjustment. This gives the following factor for the grade adjustment factor:

$$\Delta H = \frac{\pm \Sigma Q_i - Q_e}{\Sigma \frac{1}{G_i}} \tag{26}$$

where the Σ indicates the contribution from each pipe section connecting the node and these are added algebraicly with inflow positive. The numerator represents the unbalanced flowrate at the junction node.

The value of flowrate in a pipe section prior to adjustment, Q_i is computed based on the initial values of the grades at the ends of the pipe section. For sections with minor loss components this calculation is simplified if the loss coefficient for the pipe section is modified to include the minor loss term as suggested previously. The initial flowrate is

$$Q_i = (\Delta H_i/K)^{1/n} \tag{27}$$

where ΔH_i is the grade change across the pipe section based on the initial values of grade. If pumps are included then Q_i must be determined from the expression

$$\Delta H_i = K\, Q_i^{\,n} - P(Q_i) \tag{28}$$

Equ. 28 is not difficult to solve but may require the employment of an approximation procedure to solve this non-linear expression. A single trial for this method requires the adjustment of the grade for each junction node within the pipe system. The trials continue until the specified convergence criterion is met.

Simultaneous Node Adjustment (SN) Method. - This method is based on a simultaneous solution of the basic pipe network node equations and requires a linearization of these equations in terms of approximate values of the grade(13). Equ. 12 can be linearized with respect to the grades if the flowrates are written in terms of some approximate or initial values of the total grades, H_{ai} and H_{bi}, and the changes in these grades. This uses the following to calculate the flowrate in pipe section ab.

$$Q = Q_i + \frac{\partial Q}{\partial H} \Delta H_a + \frac{\partial Q}{\partial H} \Delta H_b \tag{29}$$

When the flowrate is expressed by Equ. 10, this can be solved to give:

$$Q = Q_i \left(1 - \frac{1}{n}\right) + \frac{Q_i^{1-n}}{K_{ab}} (H_a - H_b) \tag{30}$$

The initial value of flowrate, Q_i, in computed based on the initial values of the grades and is

$$Q_i = \left| \frac{H_{ai} - H_{bi}}{K_{ab}} \right|^{\frac{1}{n}} \tag{31}$$

where K_{ab} is the loss coefficient for line ab including minor losses if any.

Using Equ. 30, the continuity equation for each junction can be written as a linear function of the variable and fixed grades of the adjacent nodes and the variable grade of junction, a. This is:

$$\sum_{b=1}^{N_v} \frac{G_i^{1-n}}{nK_{ab}} H_b - H_a \sum_{b=1}^{N} \frac{Q_i^{1-n}}{nK_{ab}} = Q_e \pm \sum_{b=1}^{N} Q_i \pm \sum_{b=1}^{N_v} - \frac{Q_i}{n} - H_b \sum_{b=1}^{N_F} \frac{Q_i^{1-n}}{nK_{ab}} \tag{32}$$

Here N refers to all adjacent nodes, N_v refers to all adjacent variable grade nodes and N_F refers to all adjacent fixed grade nodes. The $\pm$ sign depends on whether Q_i is into or out of junction a and is positive for inflow.

Equ. 32 is written for each junction node in the system resulting in a set of linear equations in terms of junction node grades. For each pump in the system two additional equations are introduced which are functions of the pump suction and discharge pressure. Equ. 13 is linear and can be incorporated directly into the linearized equation set. Equ. 14 is non-linear but can be linearized using gradient methods and results in the following linearized equation:

$$H_c(1 + \beta) - H_b - H_d\beta = \alpha \tag{33}$$

Here α and β depend on the relationship used to describe the pump, P(Q), and are given by

$$\alpha = P(Q_i) + \frac{Q_i}{n} P' (Q_i) \tag{34}$$

$$\beta = \frac{P'(Q_i)}{nK_{cd}Q_i^{n-1}} \tag{35}$$

The preceeding procedure produces a set of $j + n_p$ simultaneous linear equations (where n_p is the number of pumps). These equations are solved as follows: Starting with Q_i's based on any assumed set of values for the junction node grades these linear equations are solved simultaneously for an improved set of junction node grades. These are used to compute an improved set of Q_i's and the procedure repeated until subsequent calculations satisfy a stated convergence or accuracy criterion.

EXAMPLE CALCULATIONS

The use of the five methods of analysis presented are best illustrated by an example water distribution system which is analyzed by each of the methods. The calculations are illustrated for one trial for each method. Additional trials were carried out and the solutions are compared after each method met a specified convergence criterion. The system utilized is shown in Fig. 2. All the necessary data is presented and numbers are assigned to the pipe sections and junction nodes. English units are employed for this example and appropriate conversion factors for SI units are noted on Fig. 2. This system includes one component, a globe valve in pipe no. 7, which has a significant minor loss coefficient. Other minor losses are neglected. A pump which inputs a constant power into the system is included in pipe no. 2. The useful horsepower (HP_u) for this pump is given. For this pump representation Equ. 8 becomes

$$E_p = P(Q) = Z/Q \tag{36}$$

where the pump constant $Z = 550\ HP_u/62.4 = 44.07$ for this pump. The pump terms in the various solution procedures are

$$P(Q_i) = Z/Q_i \tag{37}$$

and

$$P'(Q_i) = -Z/Q^2_i \tag{38}$$

For the example the Hazen Williams equation is employed for head loss calculations. For this expression the pipe line contant is

$$K_p = \frac{4.73\ L}{C^{1.852}\ D^{4.87}} \tag{39}$$

and $n = 1.852$ where L is the length, D is the pipe diameter. The roughness value C is defined in Fig. 2 as 130 for all the pipes. Table 1 summarizes values of pipe line, minor loss and pump constants for each pipe in the example system.

Pipe No.	K_P	K_M	Z
1	3.36	0	0
2	18.18	0	44.1
3	73.78	0	0
4	76.24	0	0
5	2.69	0	0
6	24.23	0	0
7	122.97	64.44	0

Table 1 - Pipe System Constants

These are used for all of the algorithms which are illustrated in the following sections.

<u>Algorithms Based on the Loop Equations</u>

For the example systems the loop equations include four mass continuity for the four junction nodes noted and three energy equations. The energy equations include one for each of the two loops noted (ΔE = 0) and one additional energy equation for pipes connecting the two fixed grade nodes (ΔE = 25.38f). Two of the three methods discussed for solving the loop equations require an initial set of balanced flowrates and the values used for this example are shown in Fig. 3. Values for the initial grade change, H_i, and the initial gradients, G_i, computed using Equs. 16 and 17 using the initial flowrates are shown in Table 2.

Pipe No.	Q_i	G_i	H_i
1	2	11.24	12.14
2	1.5	67.14	9.13
3	.6	88.43	28.64
4	.4	64.68	13.97
5	.5	2.76	.75
6	.1	6.31	.34
7	.4	155.88	32.84

Table 2 - Initial Values for G_i and H_i

<u>Single Path Adjustment Method</u> - A trial for this method involves adjusting each of the three energy equations using Equ. 19. The calculations for the first trial are shown in Table 3. Path AB is first adjusted using pipes 1, 2, 3 and 4. Alternate pipes connecting AB could have been chosen. The computed flow adjustment of $.178f^3/s$ is applied immediately to the pipes in path AB and subsequent calculations using

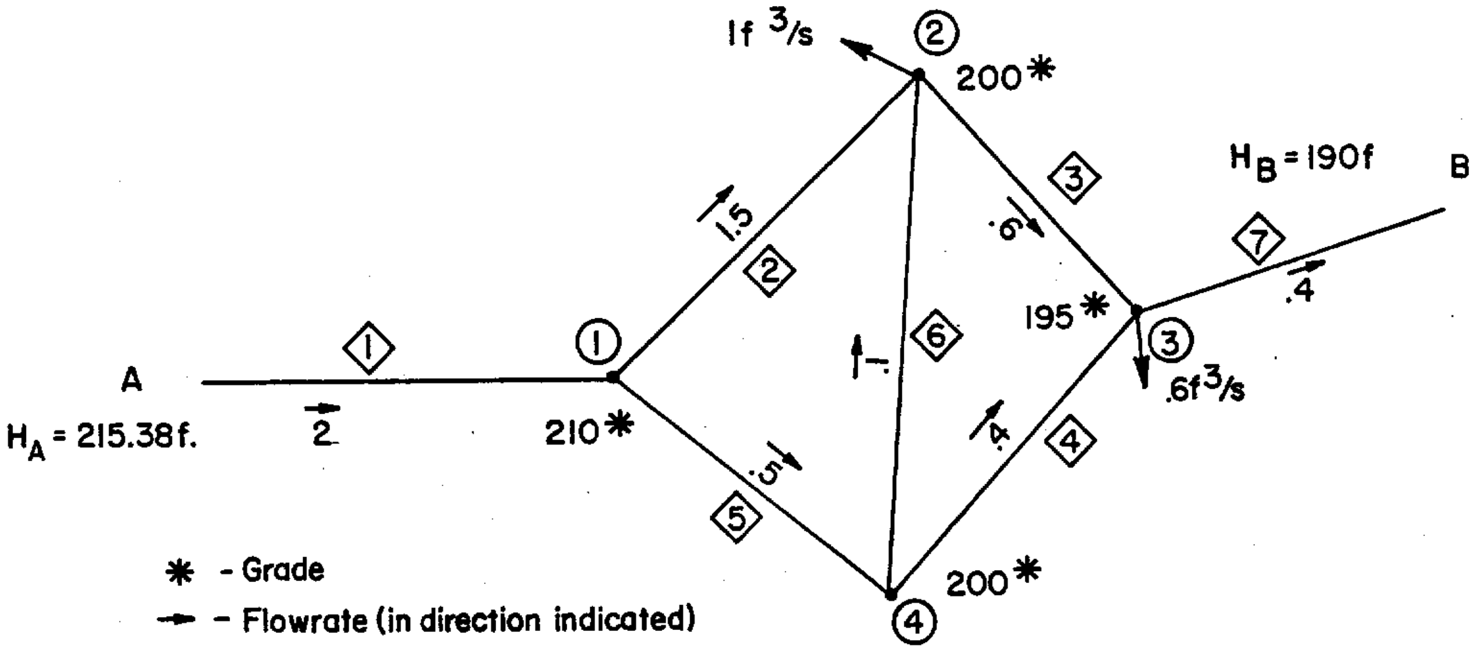

Figure 3 Initial Flowrates (in f³/s) and Grades (in f) for Example Calculations

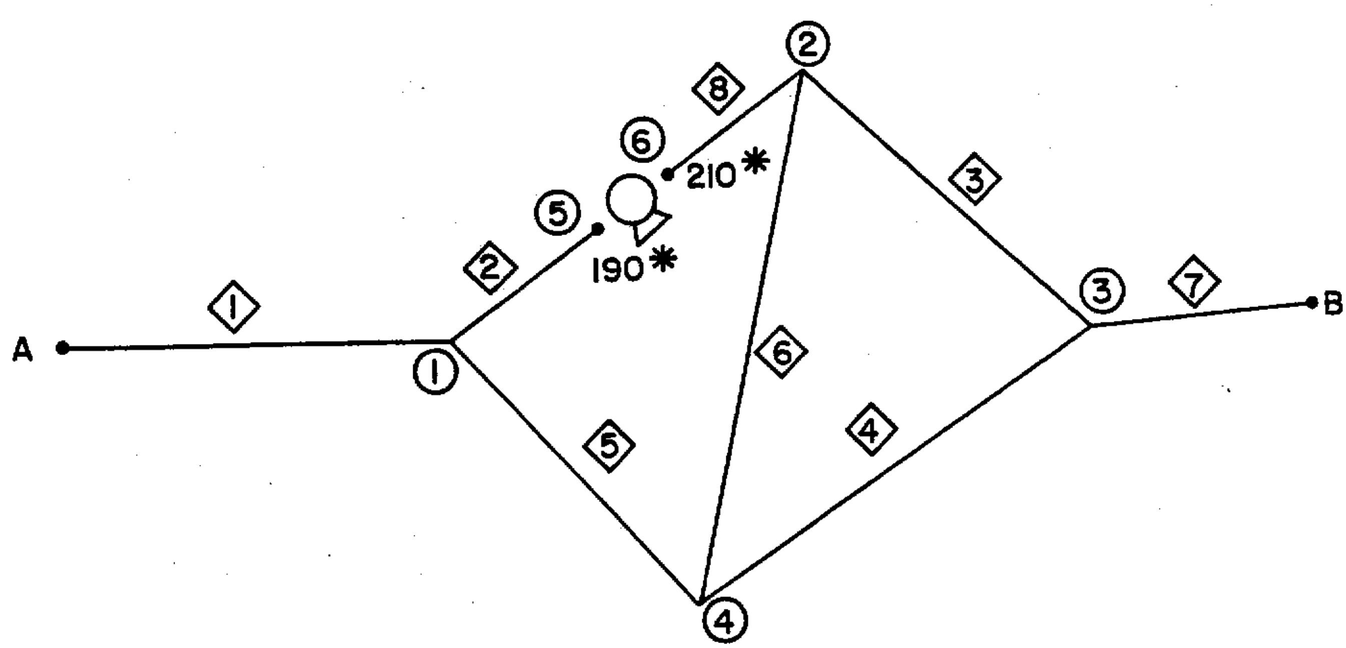

Figure 4 Pipe and Node Numbering and Initial Pump Grades (*) for Simultaneous Node Method

these pipes will enploy the adjusted flowrate Q_f, and values of G_i and H_i based on these. The second adjustment shown for loop I is carried out in the sense depicted in Fig. 3 and flows in that sense are taken as positive. The sign for the grade change, H_i, is the same as the flowrate while all the gradient terms, G_i, have positive signs. Loop II is then adjusted in the same fashion and this completes the first trial. A second trial is carried out in the same manner using the most recently determined values for flowrate for determining each flow adjustment. The trials continue until a specified convergence condition is met.

PATH AB(ΔE=25.38f)				
Pipes	Q_i	G_i	H_i	Q_f
1	2	11.2	12.1	1.82
2	1.5	67.2	9.2	1.32
3	.6	88.4	28.6	.42
7	.4	155.9	32.8	.22
		322.7	82.7	
ΔQ_{AB} = (25.38 - 82.7)/322.7 = -.178				
LOOP I (ΔE=0)				
2	1.32	67.9	-2.85	1.37
6	- .1	6.3	- .34	- .05
5	- .5	2.7	- .75	- .45
		76.9	-3.93	
ΔQ_I = -(-3.93/76.9) = .051				
LOOP II (ΔE=0)				
3	.42	65.5	14.94	.41
4	- .4	64.7	-13.97	- .41
6	.05	3.4	.09	.04
		133.6	1.06	
ΔQ_{II} = -(1.06/133.6) = -.008				

Table 3 Calculations for Single Path Adjustment Method

Simultaneous Path Adjustment Method - The application of Equ. 21 will result in three simultaneous equations for calculating the flow adjustment factors for the three energy equations (ΔQ_{AB}, ΔQ_I, ΔQ_{II}). For this example the equations are

$$23.58 - \Sigma H_i(AB) = \Sigma G_i(AB)\Delta Q_{AB} + G_i(2)\Delta Q_I + G_i(3)\Delta Q_{II}$$

$$0 = \Sigma H_i(I) = \Sigma G_i(I)\Delta Q_I + G_i(2)\Delta Q_{AB} - G_i(6)\Delta Q_{II}$$

$$0 - \Sigma H_i(II) = \Sigma G_i(II)\Delta Q_{II} + G_i(3)\Delta Q_{AB} - G_i(6)\Delta Q_I$$

The terms in parentheses denote either the path considered for a summation or the pipe considered which is common to other energy equations. The sign of terms representing contributions from common pipes is positive if the sense of the corrections are the same for both equations and negative if they are opposite. Using the values for G_i and H_i presented in Table 2 these equations become.

$$25.38 - 82.75 = 322.69\Delta Q_{AB} + 67.14\Delta Q_I + 88.43\Delta Q_{II}$$

$$0 - 8.05 = 67.14\Delta Q_{AB} + 76.21\Delta Q_I - 6.31\Delta Q_{II}$$

$$0 - 15.03 = 88.43\Delta Q_{AB} - 6.31\Delta Q_I + 159.42\Delta Q_{II}$$

and the solution is: $\Delta Q_{AB} = - .197$, $\Delta Q_I = .069$, and $\Delta Q_{II} = 0.18$.

These flow adjustment factors are applied to the initial flowrates to obtain a new set of balanced flowrates which are used for Q_i to compute the values of G_i and H_i to formulate the three equations for the second trial. The procedure continues until a specified convergence criterion is met.

Linear Method - An arbitrary set of initial flowrates is defined to start the procedure. A flowrate based on a mean flow velocity of 4f/s is used for this purpose. The initial flowrates and corresponding values for G_i and H_i are shown in Table 4. A total of four continuity equations and three energy equations are to be solved simultaneously.

Pipe No.	Q_i	G_i	H_i
1	.7854	5.072	2.151
2	.3491	375.42	-123.66
3	.1963	34.14	3.619
4	.1963	35.278	3.740
5	.7854	4.057	1.721
6	.3491	18.308	3.451
7	.1963	82.204	8.517

Table 4 Values of Q_i, G_i and H_i for Linear Method

The continuity equations (Equ. 2) are:

$$-Q_1 + Q_2 + Q_5 = 0 \quad \text{(junction 1)}$$

$$-Q_2 + Q_3 - Q_6 = -1 \quad \text{(junction 2)}$$

$$-Q_3 - Q_4 + Q_7 = -.6 \quad \text{(junction 3)}$$

$$Q_4 - Q_5 + Q_6 = 0 \quad \text{(junction 4)}$$

The energy equations (Equ. 23) using the values in Table 4 are

$$5.072\ Q_1 + 375.42\ Q_2 + 34.14\ Q_3 + 82.204\ Q_7 = 292.63 \quad \text{(Path AB)}$$

$$375.42\ Q_2 - 4.057\ Q_5 - 18.308\ Q_6 = 250.304 \quad \text{(Loop I)}$$

$$34.14\ Q_3 - 35.278\ Q_4 + 18.308\ Q_6 = 2.837 \quad \text{(Loop II)}$$

The solution of these equations (in f^3/s) is: $Q_1 = 1.725$, $Q_2 = .705$, $Q_3 = .262$, $Q_4 = .463$, $Q_5 = 1.020$, $Q_6 = .557$, and $Q_7 = .125$. These are used to formulate a second set of equations (only the energy equations change) and a second solution is obtained. The procedure continues until a specified convergence criterion is met.

Algorithms Based on Node Equations. - Methods based on the node equations solve for junction node grades and require an initial value of grade to start the solution. The values chosen are arbitary and need not satisfy any specified condition. The values used are shown on Fig.

3.

Single Node Adjustment Method - Equ. 26 is applied at each junction node to obtain a grade adjustment factor. The initial flowrates are computed from the grades at the end of the pipe section. These callculations, summarized in Table 5,

Node	1			2			3			4		
Initial Grade	210f			200			195			200		
Pipes	1	2	5	2	3	6	3	4	7	4	5	6
Q_i	1.29	-1.51	-2.03	1.38	-.23	0	.31	.23	-.15	-.29	.62	.36
G_i	7.73	67.17	9.1	67.45	39.1	0	50.4	40.4	64.6	49.2	3.32	18.8
ΔH	-8.88=-2.25/.254			3.67=.15/.041			-3.5=-.21/.06			1.84=.69/.375		
Adjusted Grade	201.12			203.67			191.5			201.84		

Table 5 Calculations for Single Node Adjustment Method

are very simple if the line contains no pump and only a loss coefficient is considered. Equ. 27 is applied in this case. If the line contains a pump Equ. 28 is solved, employing any convenient method for analyzing flow in a single pipe with a pump. Additional trials are carried out employing the adjusted grades and following the steps presented in Table 5. This procedure is continued until a specified convergence criterion is satisfied.

Simultaneous Node Adjustment Method - For the example system a total of six node equations are written. This includes equations for the four junction nodes and two additional nodes for the pump. The line containing the pump must be divided and the pump is assumed to be at the midpoint of the line. The schematic for this analysis if showed in Fig. 4. The additional pipe is numbered 8 and the additional nodes are numbered 5 (pump suction) and 6 (pump discharge). The initial grades are the same used in the previous illustration (Fig. 4) with the addition of the grades noted for the pump on Fig. 4. Based on these grades the initial flowrate in each section is easily computed (Equ. 27). These are (in f^3/s): $Q_1 = 1.24$, $Q_2 = 1.53$, $Q_3 = .23$, $Q_5 = 2.03$, $Q_6 = 0$, $Q_7 = .15$, and $Q_8 = 1.53$. With these the linearized node equations are easily written for junction nodes 1-4. These are

$$- .28H_1 + .11\ H_4 + .041\ H_5 = -26.81$$

$$- .067\ H_2 + .025H_3 + .041\ H_6 = .403$$

$$.025\ H_2 - .66\ H_3 + .025\ H_4 = -2.57$$

$$.11\ H_1 + .25\ H_3 - .134\ H_4 = .829$$

Two additional equations for the pump are:

$$H_1 + H_2 - H_5 - H_6 = 0$$

$$-\beta H_2 - H_5 + (1+\beta)H_6 = \alpha$$

For the pump description $P(Q) = Z/Q$ the terms α and β are

$$\alpha = Z(1 + 1/n)/Q_i = 44.1\ (1 + 1/1.852)/1.53 = 44.33$$

$$\beta = Z/nKQ_i^{n+1} = 44/(1.852\ 9.09\ 1.53^{2.852}) = .777$$

Here the pipe line constant K used is half the oringal value for line 2 given in Table 1. A simultaneous solution of these six linear equations gives the following values for H (in f.): $H_1 = 206.15$, $H_2 = 207.05$, $H_3 = 197.41$, $H_4 = 210.70$, $H_5 = 190.51$ and $H_6 = 222.69$. These values of grade are used to compute a new set of flowrates which are employed to formulate an improved set of equations. These are solved and the procedure continued until a specified convergence criterion is met.

Comparison of Final Results - In each situation illustrated the calculations were continued until a specified convergence criterion was met. The convergence criterion used was

$$\Sigma \left| Q - Q_i \right| / \Sigma \left| Q \right| \leq .001 \qquad (40)$$

This is applied each trial where Q is the flowrate obtained that trial and Q_i was the initial value used to carry out the calculations. This criterion roughly states that trials will continue until the relative change in flowrate between two trials is less than .1%. The results obtained for this example for each of the methods illustrated are summarized in Table 6. It can be

	PATH 8 trials	S-PATH 5 trials	LINEAR 5 trials	NODE 23 trials	S-NODE 23 trials
PIPE NUMBER	FLOWRATE →				
1	1.73	1.73	1.73	1.75	1.73
2	1.37	1.37	1.37	1.37	1.37
3	.37	.37	.37	.38	.36
4	.36	.36	.36	.37	.36
5.	.36	.36	.36	.38	.36
6	.001	.001	.001	.004	.02
7	.13	.13	.131	.14	.13
NODE NUMBER	GRADE →				
1	206.07	206.08	206.08	205.88	206.08
2	205.66	205.67	205.67	205.43	205.69
3	193.98	194.01	194.01	193.41	194.02
4	205.66	205.67	205.67	205.43	205.67

Table 6 COMPARISON OF SOLUTIONS FOR EXAMPLE
relative accuracy = .001

seen that the results are nearly identical for all the methods with only slight discrepensies for the single node adjustement method. The computer programs presented in the APPENDIX were employed for this comparison.

COMMENTS ON METHODS

The five algorithms described herein for solving pipe network problems can be readily applied to systems with a variety of configurations and containing most components including pumps. If convergence problems are not encountered, the solutions obtained by each of the methods will be identical if the number of trials carried out are sufficient.

Two methods, the single path adjustment and the single node adjustment, are suitable for hand calculations. The others require the solution of sets of simultaneous linear equations which can not be readily carried out without the aid of a digital computer.

Node equations are easier to formulate because the equations include only contributions from adjacent nodes. The loop equations require the identification of an appropriate set of energy equations which include terms for all pipes in primary loops and between fixed grade nodes. Computer formulation of this set of equations is considerably more difficult than formulation of the node equations.

Each of the procedures described requires an iterative solution and the calculations terminate when a specified convergence criterion is met. Therefore, the solutions are only approximate although the can be very accurate and represent an exact solution in some cases. The ability of an algorithm to produce an acceptable solution is of principal concern and there is significant evidence that demonstrates that convergence problems exist and an accurate solution is not always possible. The reliability of the algorithms presented in this report differ significantly and this should be considered when selecting the appropriate method. Considerable data has been obtained pertaining to the reliability of the various algorithms and this documentation will be presented.

DATA BASE FOR STUDY

A comprehensive comparison of the algorithms was possible because an extensive data base was available. These data were provided mostly by consulting engineers and water distribution engineers and represents actual or proposed distribution systems from all over the United States. In many cases these data were sent to the author because analysis difficulties were encountered. The data is summarized in Tables 7 and 8. In many cases the data include changes for additional analyses. The changes include demand changes, addition of fire demands, pipe characteristic changes, pump changes, and grade changes for fixed grade nodes (tank water surfaces elevation changes, changes in pressure level of feed lines, etc.) Table 7 summarizes the data for pipe systems under 100 pipes. There are thirty such systems with a total of sixty analyses. Table 8 summarizes the data for systems over 100 pipes. There are 21 additional systems with a total of 31 analyses.

The number of pumps, fixed grades nodes (FGN's) and changes for each system are noted in Tables 7 and 8. Some systems for which data was collected also contained check valves and pressure regulating valves. These components require special handling which differs for the various algorithms. The principal objective of the study was to evaluate the ability of the algorithms to solve the basic network hydraulic equations. The inclusion of components which require special handling may affect the reliability of the solution procedure. Therefore, systems containing pressure regulating valves and check valves are not used in the comparison and are not included in the tables.

Pumps can be described in a variety of ways using the general representation previously given

$$E_p = P(Q) \tag{41}$$

where E_p is the energy per unit weight input by the pump which is a function of the flowrate, Q. Two methods of describing pumps were utilized in this study. A constant power input was sometimes specified which gives

$$E_p = Z/Q \tag{42}$$

where Z is a function of the useful power in horsepower (or kilowatts) of the pump, P_u, and the specific gravity of the fluid, S, and is

$$\begin{aligned} Z &= 8.814\ P_u/S \qquad \text{(English units)} \\ Z &= 0.10197\ P_u/S \qquad \text{(S.I. units)} \end{aligned} \tag{43}$$

Pumps were also represented by three points of operating data as depicted in Fig. 5. For this second degree polynominal was passed through the three points to represent the pumps. This gives

$$E_p = A + BQ + CQ^2 \tag{44}$$

where A, B, and C are easily determined from the pump data used. Procedures for fitting a function to operating data provide a good representation of the pump only in the vicinity of the data. An extension of the fitted curve (dashed line) may represent the pump very badly outside the range of this data. Thus, an alternate procedure was employed for pump operations out of the range of the input data. For operational flows less than the first input flow, Q_1, a constant head operation is assumed

$$E_p = E_{p1} \quad (Q < Q_1) \tag{45}$$

For operation at flows above the third data point, Q_3, a constant power input is assumed with the power equal to that at the third data point. Thus, the pump representation employed in the comparisons for pumps described by operating data is depicted by the solid line in Fig. 5

For head loss calculations neither the Hazen Williams or Darcy Weisbach relationship was employed. Most of the data required the use of the Hazen Williams equation which is the most widely used expression for water distribution system analysis. A few of the comparisons were made using the Darcy Weisbach equation.

COMPUTER PROGRAMS FOR COMPARISONS

Computer programs were developed for each of the five algorithms studied. These programs were designed to solve the basic loop and mode equations using the calculation procedures developed and illustrated in the previously. Simplified BASIC programs are presented in the APPENDIX for each of the algorithms. These programs are intended to clearly illustrate the required computational procedures and require various input data depending on the algorithm. Data preparation instructions are included and several example systems are analyzed and compared using these programs. It can be noted that the results compare well for each example. FORTRAN programs were prepared for the actual study. These programs carried out essentially the same calculations but were considerably more sophisticated. Identical input data was used for all algorithms and this data includes the minimum required information. The connecting nodes for each pipe was the only geometic data input and no initial data was input for values of flowrates or grades. Therefore, the programs included algorithms to formulate the equations (determine pipes connecting junctions forming loops and connecting fixed grade nodes) form the connecting node data for the pipes. The programs also generated the necessary initial conditions. This required a balanced initial flowrate for the single and simultaneous path methods. An algorithm was developed that was designed to generate an arbitrary but reasonable initial balanced flowrate for these two methods. For the linear method an initial flowrate is needed but was not required to satisfy continuity. For this method an initial flowrate based on a mean flow velocity of 4f/s in an arbitrary direction was assigned. For the single and simultaneous node adjustment methods some initial values for grade are required to initiate the solution, and an algorithm was developed to provide an arbitrary but reasonable initial value. However, with pumps and other factors it is difficult to arbitrarily assign reasonable initial values for grades and flowrates so the ones assigned were sometimes in considerable error. Therefore, the calculations were often initiated with values which are significantly

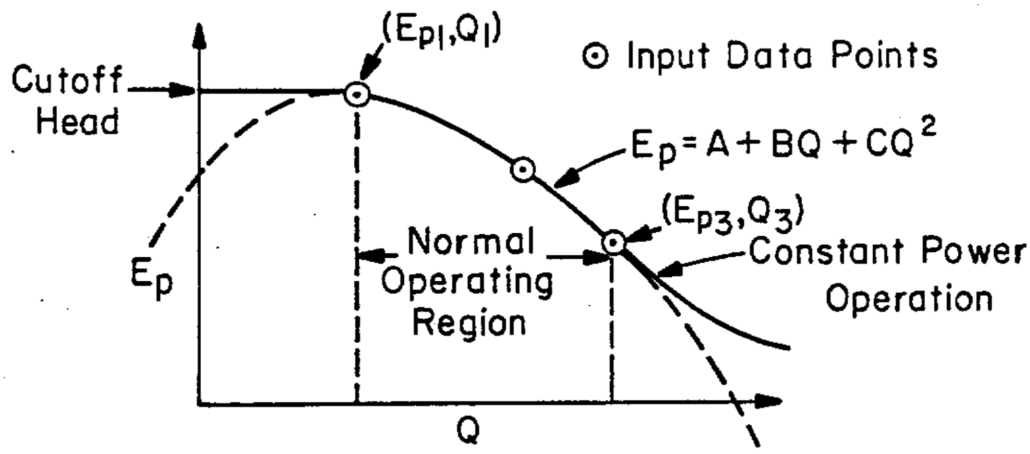

Figure 5 - Pump Operation Characteristic Curve

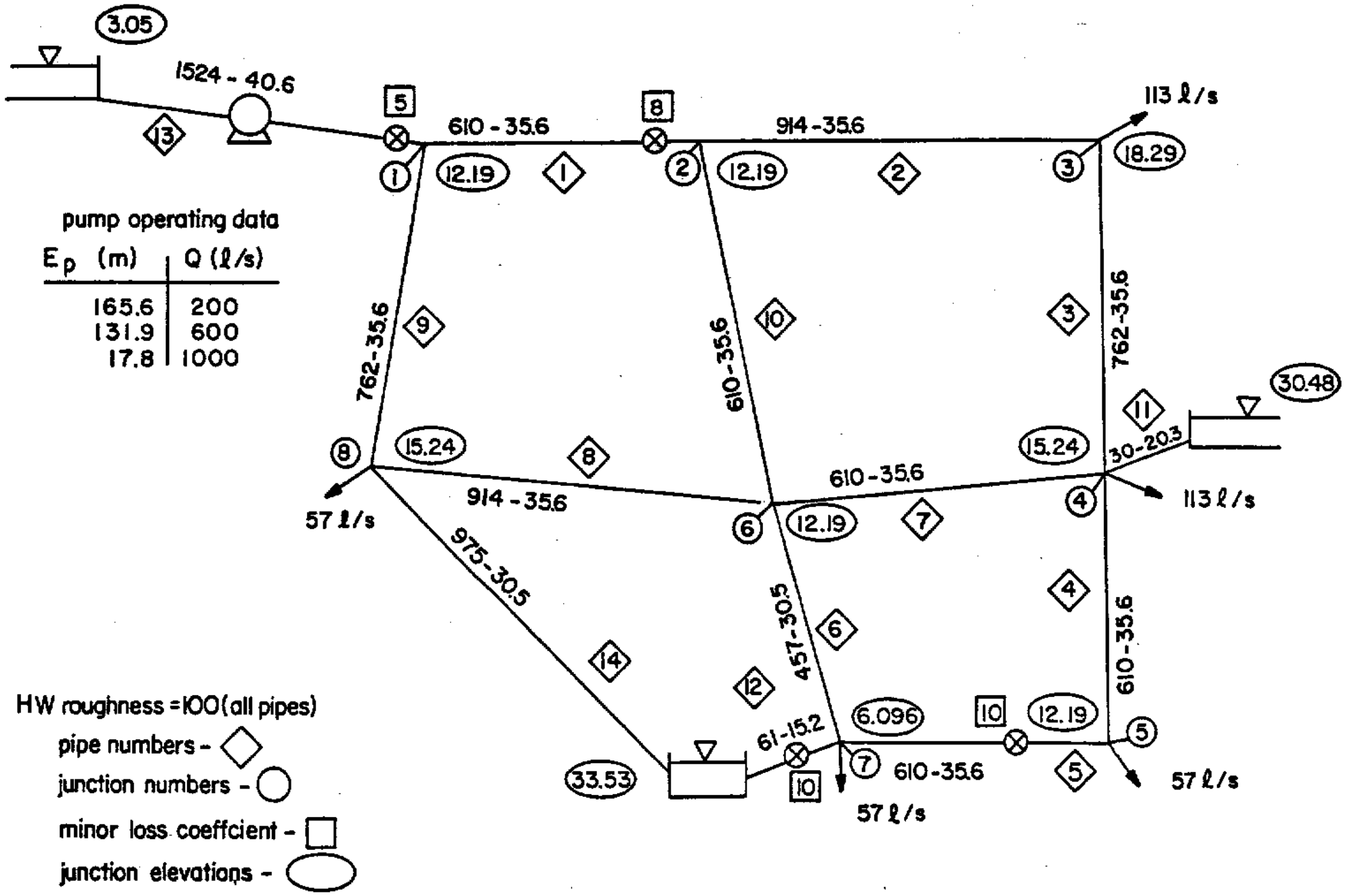

Fig. 6 Fourteen pipe system S.I. units p=14 j=8 ℓ=3 f=4 (all lengths in meters and diameters in centimeters)

different from the correct results.

The programs were designed to analyze the original input data and additional situations using data describing changes. Changes in pipeline data (dimeters, lengths, roughnesses), pump characteristics, flow demands and grades for fixed grade nodes were allowed. Systems analysed with changes are noted in Tables 7 and 8. For changes, the initial values used for flowrates or grades were the final values obtained in the previous analysis. This generally gives excellent initial values.

Convergence Criterion - For each method, the trials are continued until the specified convergence criterion was met. In this study the same convergence criterion was applied for each method. Since an updated flowrate is computed for each method the change in flowrate between successive trials was used to check convergence. The specific convergence criterion employed was

$$\frac{\Sigma\,|Q - Q_i|}{\Sigma\,Q} \leq 0.005 \qquad (46)$$

Here Q is the flowrate obtained for a trial and Q_i is the initial flowrate used which was computed from the previous trial. The numerator represents the absolute sum of the flowrate changes for a trial and this is divided by the absolute sum of the flowrates to make the criterion a more general relative condition. This criterion roughly states that when the average change in flowrates between successive trials is less than 0.5% the calculations cease. This would appear to be a quite stringent requirement which would assure good accuracy if this condition is satisfied. This convergence criterion is more stringent than ones normally applied in practice. It does not, however, assure that the flowrates are within 0.5% of the correct values. Numerous other criterion can be applied to determine the acceptability of a solution and many have been suggested and employed. Additional discussion of convergence criterion will be presented later. However, the one employed appears to be the most suitable for comparing the various algorithms on a common basis and this criterion herein is referred to as the relative accuracy.

Accuracy of Solutions - All of the methods for analyzing pipe networks yield approximate solutions. A solution is accurate when all the basic

PIPES	FGN'S	PUMPS	CHANGES
7	6	0	0
12	2	0	0
14	4	1	0
17	10	1	2
17	2	1	0
22	2	1	1
24	3	0	0
26A	7	5	0
26B	2	0	2
27	5	2	0
32	3	0	2
39	5	0	3
40A	2	0	2
40B	11	1	0
42	3	0	1
43	4	0	1
45	3	2	2
46	4	2	3
52A	3	0	2
52B	3	1	0
57	3	0	2
60	7	3	0
66	3	0	2
75	2	1	0
76	3	0	2
78	2	1	2
79	5	0	0
88	8	2	1
92	6	0	0
97	4	1	0
--			-
30(SYSTEMS)			30(CHANGES)

TABLE 7 SYSTEMS OF UNDER 100 PIPES

PIPES	FGN'S	PUMPS	CHANGES
117	3	1	0
119	2	0	0
124	14	10	0
125	13	3	0
132	4	0	2
133	8	1	1
135	4	0	0
136	7	0	1
180A	7	0	0
180B	8	0	0
189	3	1	0
198	25	6	0
213	10	4	0
225	7	0	0
235	5	11	0
254	4	0	3
280	9	0	0
305	13	0	3
381	7	2	0
400	1	0	0
509	3	3	0
--			-
21(SYSTEMS)			10(CHANGES)

TABLE 8 SYSTEMS OF OVER 100 PIPES

equations are satisfied to a high degree of accuracy. The mass continuity equations are exactly satisfied for the three methods based on the loop equations. Each of these methods are then designed to satisfy the energy equations through an iterative procedure and the unbalanced heads for the energy equations is evidence of solution accuracy. If these are satisfied to a high degree of accuracy then the solution is essentially exact. In this paper an exact solution of the loop equations will be assumed to be one where the average unbalanced head for the energy equations is less than 0.01f (.00328m). For the methods based on the node equation, iterations are carried out to satisfy mass continuity at junction nodes and the unbalance in continuity is the most significant indication of the solution accuracy.

In this study the linear method was capable of meeting the requirements for an exact solution for every situation investigated. This solution satisfies continuity exactly and the average unbalanced head in the energy equations was less than 0.01f in every case and usually much smaller. This exact solution forms a basis for comparing the results obtained by each of the methods which met the stated convergence criterion (relative accuracy ≤ 0.005).

COMPARISONS OF SOLUTIONS

All five methods were compared to an exact solution for all the situations depicted in Table 7. This involved a total of 60 comparisons and the results for each comparison were tabulated and compared as depicted in Table 9. These are the results for the 14 pipe system which is depicted in Fig. 6. The exact solution was obtained in every case by carrying out the linear method one additional trial after the relative accuracy of 0.005 was reached. This table summarizes information which is essential in evaluating the effectiveness of each algorithm. The average flowrate (138.81 ℓ/s) and grade range (30.16 m) for this solution is output. Next the flowrates and grades are compared to exact solutions for the pipes and junction nodes and average and maximum differences given for each method. The percent average and percent maximum differences are based on the average flowrate and grade range and these values are more useful for relative comparisons. Following this, additional data is given in Table 10 which further relates to the accuracy of these solutions. The unbalanced heads for each of the six energy equations for this example are summarized for the four solutions (including the exact one) obtained using the loop equations. Maximun and average values are also given. For the two solutions based on the node equations the unbalanced flows at the eight junction nodes are summarized along with maximun and average values. Percent maximun and average values were also given by dividing by the average flowrate. Finally the number of trials required and the accuracy attained for each of the six solutions is summarized. All the solutions for this example are quite good which is indicated by the good comparisons with the exact solution and also the small unbalanced heads and unbalanced flows obtained. The worst solution was attained with the single node adjustment method for which the average error in flowrate was only 1.23% and in grade was only 0.6%.

Similar comparisons were made for all 60 situations included in Table 7. Since attainment of the convergence criterion is not assured some liberal upper limits on the number of trials allowed were imposed.

THE AVERAGE FLOWRATE = 138.81

THE HEAD RANGE = 30.16

FLOWRATES

PIPE NO.	EXACT FLOWS	LINEAR	DIFFERENCE	SPATH	DIFFERENCE	PATH	DIFFERENCE	SNODE	DIFFERENCE	NODE	DIFFERENCE
1	273.35	273.35	0.0	273.36	0.01	273.31	0.04	273.36	0.01	272.60	0.75
2	149.21	149.21	0.0	149.21	0.0	149.25	0.04	149.22	0.01	149.16	0.05
3	36.21	36.21	0.0	36.21	0.0	36.25	0.04	36.22	0.01	36.72	0.51
4	1.66	1.66	0.0	1.66	0.0	1.71	0.05	-4.04	5.70	-6.87	8.53
5	-55.34	-55.34	0.0	-55.34	0.0	-55.29	0.05	-55.25	0.09	-56.40	1.06
6	95.23	95.23	0.0	95.23	0.0	95.07	0.16	95.18	0.05	95.68	0.45
7	-139.21	-139.21	0.0	-139.21	0.0	-139.24	0.03	-139.24	0.03	-141.01	1.80
8	-110.29	-110.29	0.0	-110.29	0.0	-110.26	0.03	-110.28	0.01	-110.54	0.25
9	258.24	258.24	0.0	258.24	0.0	258.23	0.01	258.24	0.0	258.44	0.20
10	124.15	124.15	0.0	124.15	0.0	124.06	0.09	124.14	0.01	124.09	0.06
11	60.76	60.76	0.0	60.76	0.0	60.79	0.03	60.72	0.04	67.61	6.85
12	-17.11	-17.12	0.01	-17.11	0.0	-17.22	0.11	-17.08	0.03	-15.78	1.33
13	531.59	531.59	0.0	531.59	0.0	531.54	0.05	531.59	0.0	530.85	0.74
14	90.95	90.95	0.0	90.95	0.0	90.97	0.02	90.95	0.0	92.30	1.35
AVERAGE DIFFERENCES			0.00		0.00		0.05		0.43		1.71
% AVERAGE DIFFERENCES			0.00		0.00		0.04		0.31		1.23
MAXIMUM DIFFERENCES			0.01		0.01		0.16		5.70		8.53
% MAXIMUM DIFFERENCES			0.01		0.01		0.12		4.11		6.15

HEADS

JUNCTION	EXACT HEADS	LINEAR	DIFFERENCE	SPATH	DIFFERENCE	PATH	DIFFERENCE	SNODE	DIFFERENCE	NODE	DIFFERENCE
1	61.49	61.49	0.0	61.49	0.0	61.51	0.02	61.49	0.0	61.73	0.24
2	40.58	40.58	0.0	40.57	0.01	40.61	0.03	40.58	0.0	40.84	0.26
3	31.86	31.86	0.0	31.86	0.0	31.66	0.0	31.86	0.0	32.06	0.20
4	31.33	31.33	0.0	31.33	0.0	31.33	0.0	31.33	0.0	31.47	0.14
5	31.33	31.33	0.0	31.33	0.0	31.33	0.0	31.34	0.01	31.45	0.12
6	36.44	36.44	0.0	36.44	0.0	36.44	0.0	36.44	0.0	36.64	0.20
7	32.41	32.41	0.0	32.41	0.0	32.40	0.01	32.42	0.01	32.53	0.12
8	41.42	41.42	0.0	41.42	0.0	41.42	0.0	41.42	0.0	41.58	0.16
AVERAGES DIFFERENCES			0.0		0.00		0.01		0.00		0.18
MAXIMUM DIFFERENCES			0.0		0.01		0.03		0.01		0.26
AVG. DIFF/HEAD RANGE-IN %			0.0		0.00		0.02		0.01		0.60
MAX. DIFF/HEAD RANGE-IN %			0.0		0.03		0.10		0.03		0.86

TABLE 9 COMPARISONS OF FLOWRATEAND GRADES FOR 14 PIPE SYSTEM

UNBALANCED HEADS

EXACT	LINEAR	SPATH	PATH
Ø.Ø	Ø.Ø	Ø.Ø	Ø.Ø2
Ø.Ø	Ø.Ø	Ø.Ø	-Ø.Ø1
Ø.Ø	Ø.Ø	Ø.Ø	-Ø.Ø5
Ø.Ø	Ø.Ø	Ø.Ø	Ø.Ø5
Ø.Ø	Ø.Ø	Ø.Ø	-Ø.Ø8
Ø.Ø	Ø.Ø	Ø.Ø	Ø.Ø3
	MAXIMUMS		
Ø.Ø	Ø.Ø	Ø.Ø	-Ø.Ø8
	AVERAGES		
Ø.Ø	Ø.Ø	Ø.Ø	Ø.Ø4

UNBALANCED FLOWS

SNODE	NODE
Ø.Ø	1.42
Ø.Ø	3.Ø2
Ø.Ø	2.94
-5.78	11.4Ø
5.79	8.55
Ø.Ø	3.71
-Ø.Ø1	1.94
Ø.Ø	1.4Ø
MAXIMUMS	
5.79	11.4Ø
% MAX. UNB. FLOWS	
4.17	6.21
AVERAGES	
1.45	4.3Ø
% AVG. UNB. FLOWS	
1.Ø4	3.1Ø

NUMBER OF TRIALS

EXACT	LINEAR	SPATH	PATH	SNODE	NODE
7	6	9	9	8	9

ACCURACY

EXACT	LINEAR	SPATH	PATH	SNODE	NODE
Ø.ØØØØ1Ø	Ø.ØØØ967	Ø.ØØØ28Ø	Ø.ØØ25Ø3	Ø.ØØ24Ø8	Ø.ØØ3465

TABLE 10. CHECKS ON ACCURACY AND CONVERGENCE - 14 PIPE SYSTEM

The limits used are

single path adjustment (P) -	200 trials
simultaneous path adjustment (SP) -	30 trials
linear (L) -	20 trials
single node adjustment (N) -	200 trials
simultaneous node adjustment (SN)	40 trials

Calculations were terminated when the accuracy of 0.005 was attained or the limit on the number of trials was reached. The abbreviations noted for each method are employed in the following discussions.

The solutions were considered to compare favorable with the exact solution if the average percent deviations from the correct solution for flowrates and grades did not exceed 10% and the maximun percent deviations did not exceed 30%. A failure was considered to have occured if the specified relative accuracy is reached and those conditions are not met or the maximum number of trials are run without attaining the specified accuracy. For all 60 situations the solutions obtained at an accuracy of 0.005 with the linear method were practically the exact solution. The largest average deviation in flowrates occured with the 79 pipe system where the average flowrate deviation was 0.1%. For the SP method convergences were also excellent with only one situation failing to reach the required accuracy and very small deviations from the correct solution were obtained. Information on failures for the SP method is summarized in Table 11. For the P method eight failures occured and these are sumarized in Table 12. In all of these cases the required accuracy was reached but the solution failed to compare satisfactorily with the correct solution. For the SN method failures were noted for 10 systems and a total of 18 situations. In the majority of cases the maximum number of trials were carried out without attaining the prescribed accuracy. Table 13 summarizes these results. For the node (N) method failures were noted in the majority of situations (51 of 60). In most of these the specified accuracy was attained but the solution did not favorably compare to the correct solution. These results are summarized in Table 14.

SYSTEM	NO. OF TRIALS	ACCURACY ATTAINED	ERROR IN FLOWRATES %AVG.	ERROR IN FLOWRATES %MAX.	ERROR IN GRADES %AVG.	ERROR IN GRADES %MAX.	UNBALANCED HEADS AVG.	UNBALANCED HEADS MAX.
52B	30	1.2085	976.8	5511.	2353.	5416.	5908.	>15,000.

TABLE 11 SUMMARY OF FAILURES FOR THE SIMULTANEOUS PATH ADJUSTMENT METHOD

SYSTEM	NO. OF TRIALS	ACCURACY ATTAINED	ERROR IN FLOWRATES %AVG.	ERROR IN FLOWRATES %MAX.	ERROR IN GRADES %AVG.	ERROR IN GRADES %MAX.	UNBALANCED HEADS AVG.	UNBALANCED HEADS MAX.
39-1	44	.00475	8.7	50.9	.34	1.16	.29	1.04
39-2	7	.00443	5.9	34.0	.22	.72	.18	.59
52B	22	.00402	31.2	166.6	28.10	63.70	8.10	92.60
66-2	8	.00443	2.4	31.4	.44	1.13	.73	1.53
76-1	13	.00474	2.8	52.2	.09	1.08	.18	.42
79	43	.00490	26.0	541.6	4.41	11.74	.27	2.08
88-1	36	.00489	5.5	30.6	.93	2.79	.50	.67
92	16	.00387	40.6	564.8	30.90	167.80	34.30	175.10

TABLE 12 SUMMARY OF FAILURES FOR THE SINGLE PATH ADJUSTMENT METHOD

SYSTEM	NO. OF TRIALS	ACCURACY ATTAINED	ERROR IN FLOWRATES %AVG.	ERROR IN FLOWRATES %MAX.	ERROR IN GRADES %AVG.	ERROR IN GRADES %MAX.	UNBALANCED FLOWS %AVG.	UNBALANCED FLOWS %MAX.
39-1	40	.05390	19.6	750.0	.0	.0	25.2	736.0
39-3	40	.01989	1.9	60.5	.0	.1	2.6	56.7
39-4	25	.00447	6.1	231.0	.0	.0	7.8	223.4
40B-1	13	.00359	2.1	53.3	2.9	3.2	3.8	56.6
45-1	29	.00411	.8	33.9	.0	.0	1.0	34.0
46-1	25	.00345	1.9	85.7	.0	.0	2.5	85.5
52A-1	19	.00179	13.6	203.0	5.4	8.9	13.9	210.6
52A-3	40	.43348	343.1	1807.2	538.4	631.5	56.1	1777.0
52B	10	.00478	112.5	651.2	159.9	237.2	3.0	48.8
57-1	40	.03011	18.7	75.2	12.1	31.2	5.1	54.0
57-2	24	.00412	15.1	61.3	13.0	36.6	3.7	30.6
57-3	1	.00488	13.7	55.7	12.0	33.3	3.7	34.0
66-1	40	.02496	2.5	61.6	.2	.3	2.6	39.4
66-3	40	.08787	533.8	23536.0	3367.0	3895.0	1190.2	20000.0
75	38	.00480	1.8	55.9	.9	1.2	2.7	58.7
88-1	40	.01160	383.5	12748.9	29.6	104.5	2.8	153.5
88-2	40	.00858	339.5	11535.2	31.5	104.5	3.0	94.2
92	8	.00462	2.6	162.9	.9	40.6	7.1	162.9

TABLE 13 SUMMARY OF FAILURES FOR THE SIMULTANEOUS NODE ADJUSTMENT METHOD

DISCUSSION AND ADDITIONAL RESULTS

The linear method proved to be the most reliable method studied. It converged in every situation to the correct solution and for all practical purposes the solution reached using a relative accuracy of 0.005 was exact. The number of trials required does not depend on the size of the system and averaged around 6 for the sixty comparisons.

The SP method also has excellent convergence characteristics and only one failure occured. For this case a constant power pump was present which operated on a very steep head-discharge curve. This was a low horsepower pump operating at a very low discharge. The steep gradient caused convergence problems for all but the linear method. This problem probably would not occur for the SP method if the pump operated on a flatter head-discharge curve. Additional trials will not result in the attainment of an acceptable solution. A second failure which was not reported occured when running the data for the 92 pipe system. The data received included some very high resistance lines where small diameters (.1 in -.254 cm) were used to stimulate closed lines. This again caused a convergence problem for all algorithms except the linear method. When the diameters were increased to a small but more reasonable diameter (.5 in) the convergence problem was eliminated for the SP method and so this data was used. In this manner the goal of producing negligible flow was accomplished without causing a convergence problem for the SP method.

The larger systems of greater than 100 pipes for which data is summarized in Table 8 were analyzed using the linear and SP methods and all attained a good solution in a reasonable number of trials. These results are summarized in Table 15 along with the required computer times. This shows that both methods obtain accurate results with few trials and use very comparable computer times. The linear method was somewhat more efficient due to better convergence characteristics resultingin fewer required trials. The costs per trial was quite similar for the two methods. This was the case in spite of the fact that the linear method requires the simultaneous solution of more

SYSTEM	NO. OF TRIALS	ACCURACY ATTAINED	ERROR IN FLOWRATES %AVG.	ERROR IN FLOWRATES %MAX.	ERROR IN GRADES %AVG.	ERROR IN GRADES %MAX.	UNBALANCED FLOWS %AVG.	UNBALANCED FLOWS %MAX.
17-1	10	.00424	10.0	41.3	61.4	63.9	9.5	21.1
17-2	34	.00482	15.3	41.9	48.4	53.8	14.2	38.2
12	49	.00488	14.4	32.5	122.0	131.0	19.5	38.0
22-1	56	.00427	28.8	71.4	21.2	30.8	16.4	88.9
22-2	3	.00447	28.5	71.4	21.1	30.8	14.8	89.3
24	26	.00434	16.6	74.3	58.8	69.0	16.8	36.1
26-1	24	.00497	8.8	39.8	9.9	37.4	4.4	21.1
26-2	47	.00456	7.9	33.1	7.3	29.2	3.4	15.8
26-3	48	.00459	16.0	57.8	13.7	33.6	5.6	26.0
26-4	23	.00398	12.9	37.7	13.7	21.0	5.8	52.6
32-1	37	.00476	13.6	69.5	32.3	38.6	12.7	33.0
32-2	2	.00400	13.0	66.7	31.2	36.9	12.0	30.2
32-3	21	.00381	7.7	34.0	26.0	34.1	8.1	20.0
39-1	29	.00393	8.2	75.2	.3	.7	5.0	42.3
39-2	9	.00470	7.0	157.6	.1	.3	7.6	160.3
39-3	28	.00460	8.3	45.8	.8	3.5	4.4	21.9
39-4	17	.00486	7.0	48.1	.4	1.1	3.7	23.6
40B-1	132	.00426	49.7	193.3	124.5	136.2	44.2	312.9
40B-2	91	.00471	30.9	149.0	86.7	93.0	24.7	203.6
40B-3	200	.05109	56.2	266.7	323.6	340.3	39.0	254.0
42-1	120	.00379	24.8	104.2	12.8	16.0	11.1	77.8
42-2	147	.00391	17.6	57.8	30.0	35.3	10.4	37.8
45-1	33	.00362	17.2	103.0	8.7	17.4	12.0	33.8
45-2	42	.00389	18.2	128.7	5.9	11.3	14.5	178.3
45-3	11	.00414	34.0	285.4	20.7	36.5	21.2	195.4
46-1	35	.00452	7.7	71.1	1.8	6.7	5.5	29.6
46-2	21	.00492	10.4	76.3	2.6	9.4	6.2	38.1
46-3	18	.00494	12.2	74.0	2.0	8.4	6.8	34.6
46-4	11	.00499	6.3	42.4	1.3	4.7	4.0	30.3
52A-1	200	.02585	1564.0	5612.8	7414.0	9231.0	420.1	3375.0
52A-2	200	.01796	200.0	804.0	333.4	426.0	58.2	554.0
52A-3	84	.00464	349.4	1357.8	324.7	423.6	83.4	700.5
52B	67	.00460	36.6	164.9	62.2	83.6	20.0	75.1
57-1	200	.02841	1098.0	4500.0	4226.0	5551.0	328.1	2567.3
57-2	200	.02146	769.0	3130.0	2415.0	4433.0	250.0	1991.0
57-3	200	.02103	568.8	2249.0	1463.0	3023.0	166.9	1181.1
60	99	.00404	31.9	153.8	71.0	96.2	15.9	102.6
66-1	76	.00397	138.8	1242.0	77.0	98.0	85.2	1218.1
66-2	133	.00485	57.8	497.4	25.1	44.3	31.0	471.2
66-3	118	.00437	49.6	549.7	10.8	33.5	36.1	592.0
75	41	.00462	21.4	147.3	11.0	15.8	20.9	79.8
76-1	142	.00469	163.1	1067.0	269.3	438.3	44.7	243.4
76-2	200	.03175	44.7	306.9	44.9	61.0	15.8	93.1
76-3	133	.00409	31.4	223.6	37.5	47.8	12.1	54.6
78-1	20	.00418	23.5	129.1	53.7	75.3	8.7	44.3
78-2	11	.00274	13.3	103.3	22.7	31.4	4.7	24.3
78-3	14	.00432	20.8	121.1	88.5	119.3	6.9	65.8
79	131	.00348	16.6	84.0	11.7	21.4	9.8	56.8
88-1	87	.00484	23.2	261.4	5.8	8.4	13.1	91.9
88-2	5	.00476	4.6	40.2	.6	2.0	5.2	62.2
92	33	.00463	12.6	96.8	14.1	62.0	5.8	116.3

TABLE 14 SUMMARY OF FAILURES FOR THE SINGLE NODE ADJUSTMENT METHOD

equations than the SP method. However the basic equations contain mostly zero terms and the total non zero terms for the two methods does not differ greatly. If sparse matrix methods which deal only with the non zero terms are employed to solve the linear equations then very comparable computer times per trial are required for the two methods. Sparse matrix routines were employed in this study. If full matrix methods are employed then the SP method would be faster.

The three other methods studied exhibited significant covergence problems, and the frequency of problems increases as larger systems are analyzed. Since adequate documentation of convergence problems were obtained for these methods using systems of leff than 100 pipes, results for the larger systems were not compared for the P, N and SN methods. A number of these larger systems were analyzed with these methods and a significant number of convergence problems were encountered. However, excessive computer times would be required to completely document these results so the comparisons for these methods were limited to the smaller systems.

The eight failures obtained for the P method all reached the specified accuracy. In addition, with the exception of the 52B and 92 pipe systems all the solutions attained a relatively low average unbalanced head for the energy equations with an average unbalanced head for these six cases of 0.37f. Yet significant errors exist in these solutions. These eight cases were rerun with relative accuracy of 0.0005 specified, and with additional trials, all attained this. The results are summarized in Table 16. Five of the cases improved to an acceptable degree. The three failures include the 52B system with the steep pump curve and the 92 pipe system with high resistance lines, and the failures are due to these factors and evidenced by the large unbalanced heads. However, the one additional failure was documented for the 79 pipe system which by all indications reaches a very good solution. The average unbalanced head is only 0.06f. Yet many flowrates are in great error. This was especially evident for flow into and out of storage tanks and these results are presented in Table 17.

It can be seen that for each of the three solutions the computed net inflow into the system was 2350 gpm but the predicted distribution from the tanks is in great error for tanks A and B. Since this

NO.OF PIPES	L METHOD NO. OF TRIALS	L METHOD TIME (SECS)	SP METHOD NO. OF TRIALS	SP METHOD TIME (SECS)
117	7	2.0	7	1.9
119	7	2.2	8	2.3
124	8	2.7	9	4.0
125	7	2.3	22	6.1
132-1	8	5.1	10	5.5
132-2	4		4	
132-3	4		5	
133-1	4	2.5	7	2.7
133-2	3		5	
135	7	2.3	9	2.6
136-1	8	6.4	7	8.0
136-2	4		4	
180A	6	2.8	6	2.7
180B	5	3.9	7	4.5
189	10	5.5	9	5.2
198	5	3.5	9	7.1
213	7	5.0	14	6.0
225	6	7.0	10	10.0
235	6	6.2	8	8.1
254	7	9.8	9	10.0
280	6	9.7	7	11.1
305	6	11.3	8	15.5
381	6	11.1	7	15.4
400	8	15.5	9	22.0
509	6	17.6	12	22.8
AVERAGE	6.4	5.38	8.5	6.94

TABLE15 RESULTS FOR L AND SP METHODS
SYSTEMS OF MORE THAN 100 PIPES

SYSTEM	NO. OF TRIALS	ACCURACY ATTAINED	ERROR IN FLOWRATES %AVG.	ERROR IN FLOWRATES %MAX.	ERROR IN GRADES %AVG.	ERROR IN GRADES %MAX.	UNBALANCED HEADS AVG.	UNBALANCED HEADS MAX.
39-1	88	.000496	.9	5.3	.04	.12	.03	.11
39-2	13	.000408	.2	1.3	.01	.04	.01	.04
52B	263	.000499	10.9	59.4	12.30	18.90	2.80	28.50
66-2	22	.000480	.2	3.0	.05	.13	.07	.15
76-1	30	.000474	.2	4.0	.01	.15	.02	.05
79	163	.000500	16.8	377.1	2.30	4.20	.06	.70
88-1	67	.000468	.5	5.4	.06	.20	.07	.57
92	23	.000498	40.6	564.8	31.00	169.50	34.10	175.90

TABLE 16 RESULTS FOR THE SINGLE PATH ADJUSTMENT METHOD-HIGHER ACCURACY(.0005)

	CORRECT SOLUTION	SOLUTION FROM P METHOD (.005)	SOLUTION FROM P METHOD (.0005)
FLOW FROM SOURCE	812.2 IN	811.6 IN	813.5 IN
TANK A	175.7 IN	626.9 OUT	385.5 OUT
TANK B	91.5 IN	897.4 IN	632.5 IN
TANK C	399.2 IN	419.9 IN	405.2 IN
TANK D	871.3 IN	848.1 IN	884.1 IN

TABLE 17 RESULTS FOR TANK FLOWS - 79 PIPE SYSTEM - FLOWS IN GPM

TRIAL NO.	RELATIVE ACCURACY	AVERAGE CHANGE IN HEAD	RELATIVE AVERAGE UNBALANCED FLOW
1	6.868085	2.062292	0.224693
2	1.172656	0.714327	0.717489
3	0.304951	0.290744	0.435035
4	0.590466	0.196319	0.610953
5	0.170562	0.109673	0.393212
6	0.341035	0.042137	0.519106
7	0.145728	0.014454	0.304546
8	0.339205	0.094311	0.428954
9	0.245958	0.031332	0.241429
10	0.219468	0.028544	0.147335
11	0.077652	0.002830	0.062965
12	0.035240	0.043724	0.057031
13	0.111314	0.014584	0.055129
14	0.050563	0.005066	0.040292
15	0.023733	0.066776	0.046307
16	0.097162	0.022956	0.032567
17	0.030619	0.037164	0.039966
18	0.065778	0.009679	0.032403
19	0.049518	0.019692	0.091392
20	0.055300	0.011937	0.071025
↓	↓	↓	↓
181	0.042277	0.026720	-0.080546
182	0.056857	0.025674	0.044164
183	0.063956	0.026278	0.060448
184	0.060892	0.036050	0.067433
185	0.060131	0.026682	0.048817
186	0.050636	0.039783	0.049264
187	0.064525	0.054600	0.047559
188	0.086900	0.028453	0.050333
189	0.049111	0.032251	0.042745
190	0.047099	0.016108	0.040846
191	0.032896	0.053125	0.041617
192	0.082833	0.029083	0.042004
193	0.062752	0.022287	0.038893
194	0.027142	0.017497	0.026566
195	0.033371	0.012186	0.036093
196	0.024823	0.024109	0.034614
197	0.046499	0.015606	0.033525
198	0.036479	0.033087	0.037066
199	0.082149	0.033339	0.102533

TABLE 18 DOCUMENTATION OF CONVERGENCE PROBLEM - SN METHOD

prediction is of significant engineering importance the results from the P method are unacceptable. This is in spite of the very high degree of relative accuracy attained and the low value of unbalanced head reached. All indicators point to an excellent solution when, in fact, it is not. The reason that this happens is that some lines with high and low head losses are included in the same energy equations. This is not a highly unusual in water distribution systems and yet the P method encounters difficulty with this situation.

When the SN method is successful a highly accurate solution is obtained in relatively few trials. However, a significant number of failures were summarized in Table 13 for this method. The patterns of failures is the most varied of the methods investigated. Six of the 18 failures noted attained an extremely accurate calculation of the grades. In these cases the calculation of most flowrates were also accurate. However, a few exhibit large errors. This occurs when lines which carry significant flows at very low head losses are present. This is not uncommon in many water distribution systems and occurs frequently for analyses made during periods of slack demands. Lines to storage tanks often are low resistance lines and small errors in grade calculations can result in highly inaccurate calculations of flowrate in these lines. A second type of failure is one where the specified accuracy is not attained regardless of the number of trials carried out. Failures of this type have been noted for this method and previously published references to this type of convergence difficulty were cited (8, 9, 10). Table 18 gives some typical results for this situation which show that after a point the relative accuracy no longer improves but tends to oscillate, and an acceptable solution is never attained even though 200 trials are completed. The average change in head and average unbalanced flow also exhibit the same tendency as show in Table 17. This behavior which is illustrated for the 57 pipe system, was exhibited frequently. For two systems, failures of this type occured for several situations at the same time that a very accurate solution was obtained for other situtations. For the 66 pipe system a convergence failure occured for the first situation. A second situation where a single additional inflow representing a new storage tank was added and an excellent solution was obtained in four trials which satisfied the required relative accuracy and had an average error in flowrate of 0.18%. A third situation where this inflow as reduced by half was analyzed starting with very good values of grades (those just obtained) and yet a convergence failure occured with a few extremely poor results. A similar situation occured with the 52A pipe system where an accurate solution was obtained for the second of three situations while the other two failed. In this case the third situation introduced a pump which produces a more significant change and led to a severe convergence problem. These results document the very sensitive behavior of this algorithm.

A number of situations that failed for the SN method were rerun with more trials allowed or with a higher accuracy spcified or both and these results are summarized in Table 19. Six of the nine cases still could not meet the convergence condition after 400 trials. Only one system (39 pipes) improved due to additional trials. A second system (75 pipes) which met the original relative accuracy specification but was not an acceptable solution, was run with higher accuracy (0.0005) and did not attain the specified accuracy. The maximum allowable trials were rerun and an improved solution was attained which met the required standards. Failures for the SN method are evidenced by relatively high values for unbalanced flows at junctions, and this is probably the most reliable indicator for this method.

Based on the criterion for a successful solution the N method was the least reliable of the methods studied. Although the specified accuracy was reached in most cases, the solution failed to meet the required standards. It is not summarized in Table 14 but the solutions which failed often reached a very low value for the average head change between trials which indicates that the grade adjustments were very small. For example the 40 pipe system which failed for three situations reached an average grade change of 0.03f along with a relative accuracy of less than 0.005, and yet the solutions contained very large errors. In many of the situations listed in Table 14 average grade changes of less than 0.1f were attained. It appears that neither the relative flowrate accurcy or the average head change is a reliable indicator of an acceptable solution for this method. The relative average and maximum unbalanced flowrates at the junction nodes are more reliable

SYSTEM	NO. OF TRIALS	ACCURACY ATTAINED	ERROR IN FLOWRATES %AVG.	%MAX.	ERROR IN GRADES %AVG.	%MAX.	UNBALANCED FLOWS %AVG.	%MAX.
39-1	104	.00306	.8	12.6	.0	.0	1.1	9.1
39-3	14	.00481	.7	11.8	.0	.0	.8	9.1
52A-1	400	.06810	14.6	50.7	8.2	11.1	2.6	24.3
52A-3	51	.00048	66.9	274.7	39.5	86.1	6.4	236.0
57-1	400	.06270		NO IMPROVEMENT				
57-2	400	.02414		NO IMPROVEMENT				
57-3	400	.09877		NO IMPROVEMENT				
75	400	.00382	1.2	13.2	1.2	1.6	1.1	12.5
88-1	400	.01141		NO IMPROVEMENT				

TABLE 19 RESULTS FOR THE SIMULTANEOUS NODE ADJUSTMENT METHOD WITH ADDITIONAL TRIALS (AND/OR HIGHER ACCURACY)

SYSTEM	NO. OF TRIALS	ACCURACY ATTAINED	ERROR IN FLOWRATES %AVG.	%MAX.	ERROR IN GRADES %AVG.	%MAX.	UNBALANCED FLOWS %AVG.	%MAX.
26-1	70	.000464	3.1	16.8	4.4	18.3	1.6	11.6
26-2	105	.000427	4.3	16.4	4.4	18.6	1.5	14.2
26-3	128	.000486	7.0	23.9	6.9	18.9	1.9	11.0
26-4	84	.000482	1.4	4.4	1.6	2.6	.7	4.1
39-1	56	.000461	1.6	16.1	.1	.2	.9	5.5
39-2	63	.000010	.3	6.2	0.0	.0	.3	5.8
39-3	62	.000422	.8	5.8	.0	.2	.5	3.2
39-4	43	.000468	1.1	6.5	.1	.2	.5	2.9
46-1	83	.000339	.7	8.9	.2	.6	.7	6.2
46-2	91	.000446	1.2	8.9	.3	.9	.6	2.5
46-3	51	.000448	1.5	12.3	.2	.9	.9	9.0
46-4	73	.000453	1.2	9.4	.3	1.1	.6	4.5
79	266	.000471	3.3	14.4	2.4	4.6	1.7	24.4
88-1	237	.000477	4.1	58.0	.7	1.2	2.6	27.6
88-2	117	.000476	4.7	62.2	1.7	2.8	2.7	30.8

TABLE 20 RESULTS FOR THE SINGLE NODE ADJUSTMENT METHOD FOR SELECTED SYSTEMS- ACCURACY= .0005

SYSTEM	NO. OF TRIALS	ACCURACY ATTAINED	ERROR IN FLOWRATES %AVG.	%MAX.	ERROR IN GRADES %AVG.	%MAX.	UNBALANCED FLOWS %AVG.	%MAX.
52A-1	1000	.016910		NO IMPROVEMENT				
52A-2	1000	.008600		NO IMPROVEMENT				
52A-3	536	.000424		NO IMPROVEMENT				
57-1	1000	.009510		NO IMPROVEMENT				
57-2	1000	.004540		NO IMPROVEMENT				

TABLE 21 RESULTS FOR THE SINGLE NODE ADJUSTMENT METHOD FOR SELECTED SYSTEMS- ADDITIONAL TRIALS

indications.

A number of the failures note in Table 14 for the N method did appear to be converging although the solutions did not meet the required standard satisfactorily. It appeared that an acceptable solution for the N method could be attained in these situations if more trials were attempted. Some selected systems which reached the specified accuracy of 0.005 in a reasonable number of trials were run at an accuracy of .0005, and in all but one case (two situations) a good solution was attained. These results are summarized in Table 20. The solution for the 88 pipe system still failed to meet the required conditions with relative flowrate errors of around 60% documented. The average unbalanced flows were around 2.7% which seems very reasonable. It appears that most of the solutions which failed at a relative accuracy of 0.005 but reached that accuracy in a reasonable number of trials could be improved to provide an acceptable result if a more stringent and perhaps different convergence criterion is met. Convergence may be slow, however, and it may be difficult to provide assurance that the solution is a good one. In the cases where the relative accuracy of 0.005 was not met in the allowable limit of trials (200), additional trials did not tend to improve the solution even if a more stringent relative accuracy could be met. Table 21 summarizes a few situations where were run up to 1000 trials with very poor results. This implies that the N method is not capable of solving these systems regardless of the number of trials carried out. It appears that the most significant factor adversly affecting convergence of the N method is the presence of low resistance lines.

CONCLUSIONS

Significant convergence problems were documented for the P, N, and SN methods. These methods are widely used and the results of this study indicate that greatcare must be exercised when employing these methods. The single adjustment methods (P and N) which are usually employed if a hand solution or a small computer is used must be carried out to much greater accuracy than normally called for to increase the probability that a good solution is attained. Attainment of a relative accuracy of 0.0005 with these methods greatly reduce the chances of failure. Stringent convergence requirements for unbalanced heads for the P method and unbalanced flows for the N method also may be employed to improve reliability. However, the attainment of a stringent convergence criterion does not assure that the solution is accurate and a significant number of situations were documented for both methods where substantial and serious errors occured after meeting stringent convergence criterion. A few situations were documented where the specified accuracies could not be attained and the solutions were not acceptable. It is concluded that if a specified stringent convergence criterion cannot be met using single adjustment methods, the solution is not reliable.

Failures for the SN method were characterized by the inability to meet a reasonable convergence criterion and if this occurs in a limited number of trials (40 in this study) additional trials are usually of no benefit. The failure rate was quite high with this method and the use of results obtained using this method is not recommended unless a good accuracy is attained in a reasonable number of trials. The best indication of an acceptable solution appears to be the average relative unbalanced flow at the junction nodes. The indications are that this value should be less than 2%.

Each of the three methods which experienced significant convergence problems requires a set of flowrates or grades to initate the solution and failure can be reduced if initial values are employed which are closer to the correct values. However, there appears to be no reliable means of consistently determining better initial values. In addition, starting with an excellent set of initial conditions does not assure convergence as evidenced by failures for situations starting with solutions for similar situations. In some cases where line losses vary greatly or pumps operate on steep curves the correct solution can not be attained unless the initial values are very close to the correct ones and this clearly is not possible. Many of the reliability problems documented using algorithms based on the node equations (N and SN methods) are due to the inability of these methods to handle low resistance lines. For these lines small errors in grade calculations. The inability to handle this situation is inherent for algorithms based on node equations and this is due to the fact that solution algorithms for these equations do not incorporate an exact continuity balance.

Both the SP and L methods provide excellent convergence where flowrates and grades are computed with great accuracy and the attainment of a relative flowrate accuracy of 0.005 is adequate to assure this. There is, however, no absolute assurance of convergence, although convergence failure will be rare using the SP method and occur less frequently using the L method. The situations included in this study represent a wide variety of systems, some of which incorporate features which increase convergence difficulties. The SP and L methods attained very accurate solutions in relatively few trials with only one failure noted for the SP method. However, since gradient methods are used to handle non linear terms, convergence problems are always a possibility, particularly if ill conditioned data such as poor pump descriptions are employed. The author has had numerous contacts with engineers using the L method and has no evidence of convergence difficulties if reasonable system data is employed.

It is concluded that, if possible, either the SP or L method should be employed for pipe network analysis and that convergence is virtually assured if reasonable data is employed. Of the two methods the L method has slightly better convergence characteristics and does offer some advantages. A balanced initial set of flowrates is not required for the L method since the continuity conditions are incorporated into the basic set of equations. This also allows a more direct and reliable incorporation of hydraulic components such as check valves, closed lines and pressure regulating valves. These components were not included in the systems studied but must be handled by a pipe network analysis algorithm if it is to be of general use. These features affect continuity and their effect can be incorporated into the basic set of equations solved by the L method. The SP method requires less direct methods for these components since only the energy relationships are solved and the initial continuity balance must be maintained. The SP method does require the solution of significantly fewer equations which will be a benefit if full matrix methods are employed. If sparse matrix methods dealing only with non zero terms are employed this benefit is lost and the L method is somewhat more efficient as demonstrated in this study.

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The following symbols are used in this paper:

A = line area

C = Hazen Williams Roughness Coefficient

D = line diameter

$E_{p'}$ = energy input by pump

f = number of fixed grade nodes

g = gravitational constant

G_i = gradient for head change based on approximate flowrate

h_{LM} = energy loss due to minor losses

h_L = energy loss in pipe section

h_{LP} = energy loss due to wall shear

H = grade (head) at junction node

H_i = head change in pipe section based on approximate flowrate

HP_u = useful horsepower for pump

j = number of junction nodes

K = loss coefficient for pipe section

K_M = loss coefficient due to minor losses

K_p = loss coefficient due to line loss

ℓ = number of primary loops

L = line length

M = minor loss coefficient

n = exponent for head loss expression

n_p = number of pumps in system

N = number of nodes adjacent to given node

N_F = number of FGN's adjacent to given node

N_V = number of variable grade nodes adjacent to given node

p = number of pipes in system

$P(Q)$ = function for pump description (depends on flowrate, Q)

Q = flowrate in pipe section

Q_e = enternal flowrate at junction node

Q_f = flowrate in pipe section after adjustments

Q_i = flowrate in pipe section prior to adjustments

Q_{in} = flowrate into junction node

Q_{out} = flowrate out of junction node

Z = constant for constant power pump description

α = constant for pump description - SN method

β = constant for pump description - SM method

ΔE = constant grade difference for energy equation

ΔH = grade adjustment for N and SN methods

ΔQ = flow adjustment for P and SP methods

The following subscripts are used for flowrates, Q, grades, H, and loss coefficients, K:

a, b, c, d

j,k

1, 2, 3, 4, 5, 6, 7, 8

APPENDIX

Computer Programs - Common Information

The computer programs which are appended were written to illustrate the application of the various algorithms to pipe network analysis. Each program is written assuming that for a system of P pipes and J junction nodes, the pipes are numbered from 1 to P and the data is input in this order and the junction nodes are numbered from 1 to J and input data for junction nodes, if required, is input in this order. Some notation is common to the five programs. This is summarized below. The first eight items are input data for all programs.

P	- number of pipes in pipe system
J	- number of junction nodes in system
C1	- flow conversion factor (f^3/s to units used)
L1	- length of pipe section (ft)
D1	- diameter of pipe section (in.)
R1	- Hazen Williams Roughness Coefficient
M1	- sum of minor loss coefficients in pipe section
P1	- useful horsepower for pump
X	- exponent for head loss equation - 1.852
D(N)	- diameter fo pipe N in feet (D1/12)
Z(N)	- pump energy term for pipe N - P1(550/624)
Q(N)	- flow rate in pipe N
H(N)	- grade at junction node N
Q8	- sum of absolute changes in flowrate between trials
Q9	- sum of absolute flowrates for trial
Q5	- relative change in flowrate between trials
T5	- trial counter

Additional notation will be defined for each sample program. The input data requirements depend on the algorithm used and are summarized for each program.

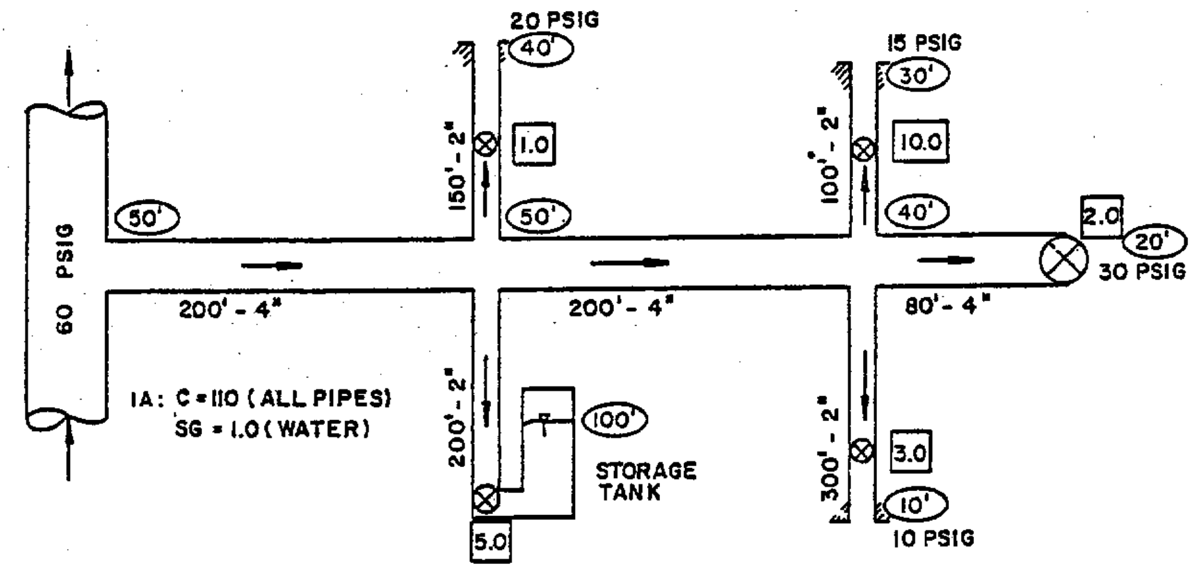
Each program uses the same criterion for terminating the calculations and this occurs when the relative change in flowrate (absolute) between the last two trials is less than .1 percent. This is

$$\frac{\Sigma\ |Q(N)_i - Q(N)_{i-1}|}{\Sigma\ |Q(N)_i|} < .001$$

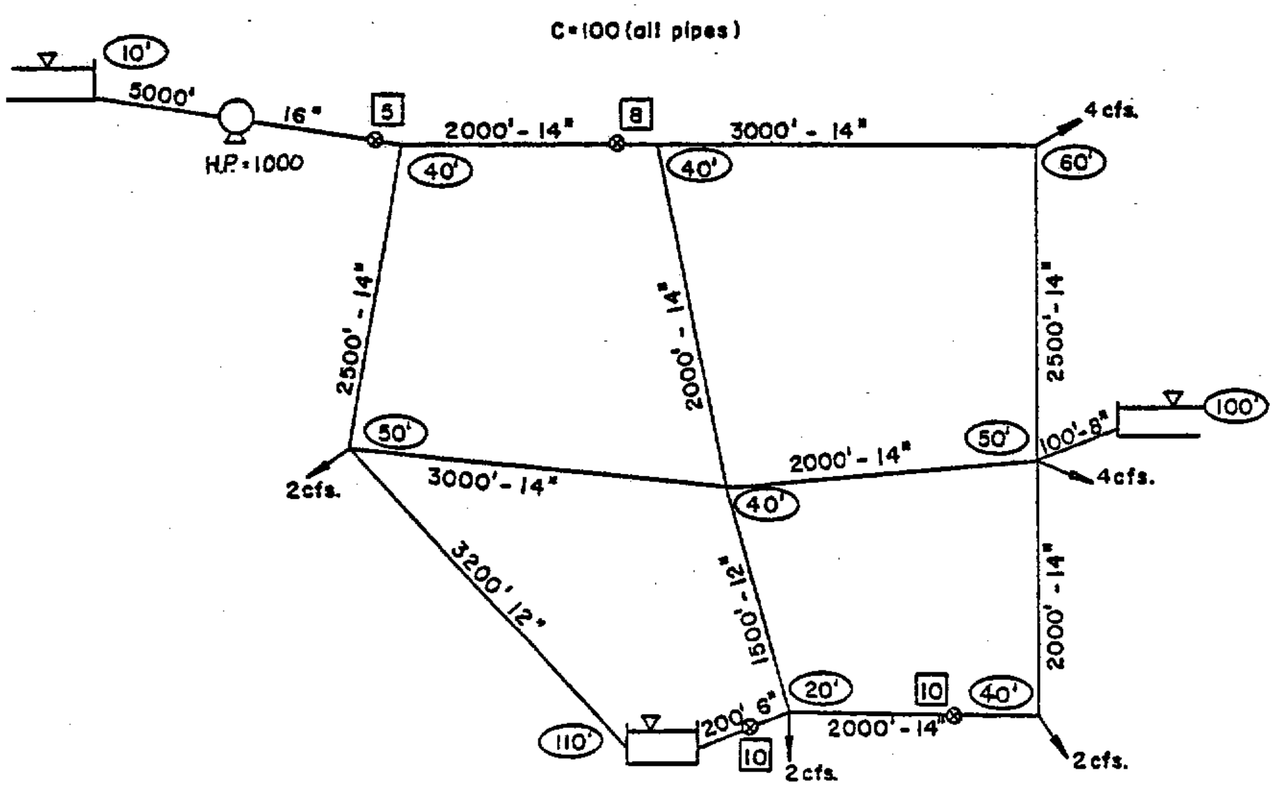
where i refers to the last trial and i-1 to the next to the last trial.

Each program is used to analyze five examples which appear in the USERS MANUAL and the flowrates and grades obtained for these examples are output along with an indication of the convergence characteristics of each solution as the solution trials are carried out. Schematics for these examples were also appended. For systems with pumps, useful power is given.

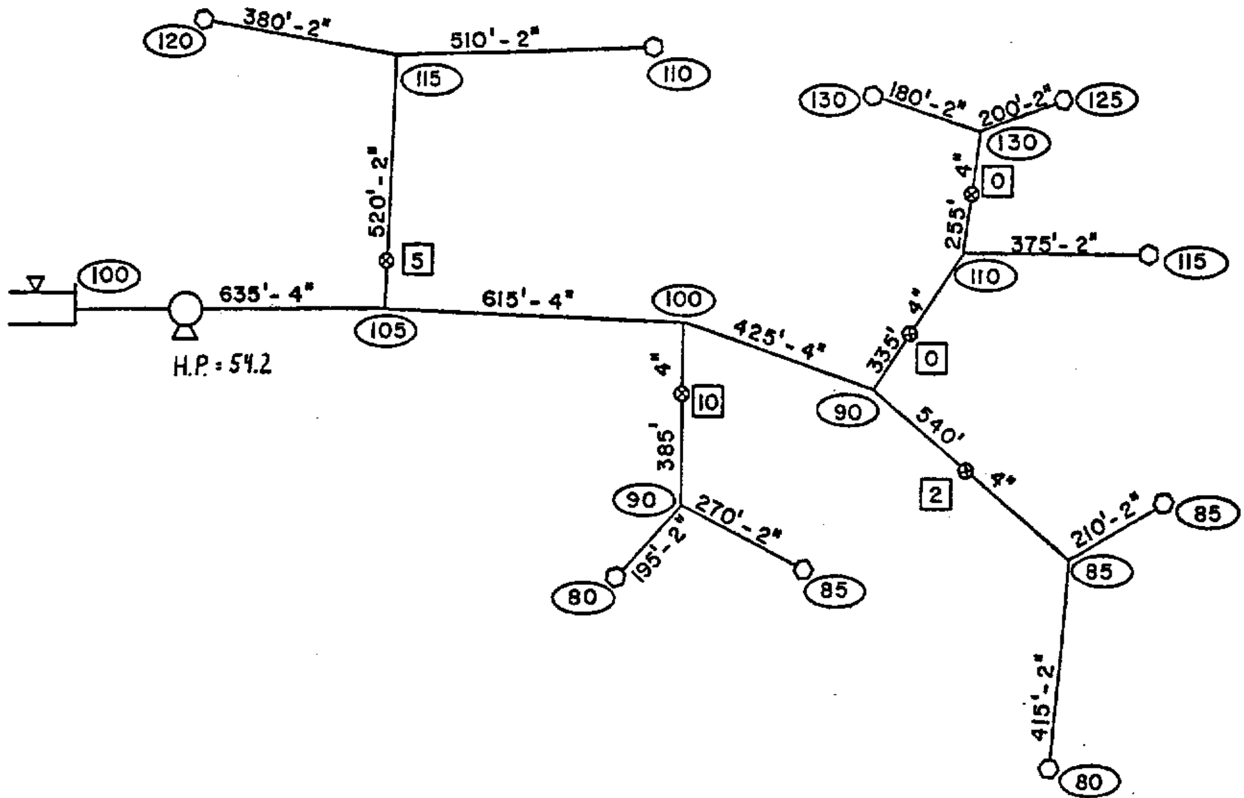
The solution obtained by the various algorithms are in quite good agreement although some discrepancies occur. The solutions obtained by the linear method are the most accurate ones.



Example 1 Distribution system fed by large pressure main – required discharge pressures specified. p=7, ℓ=0, j=2, t=6



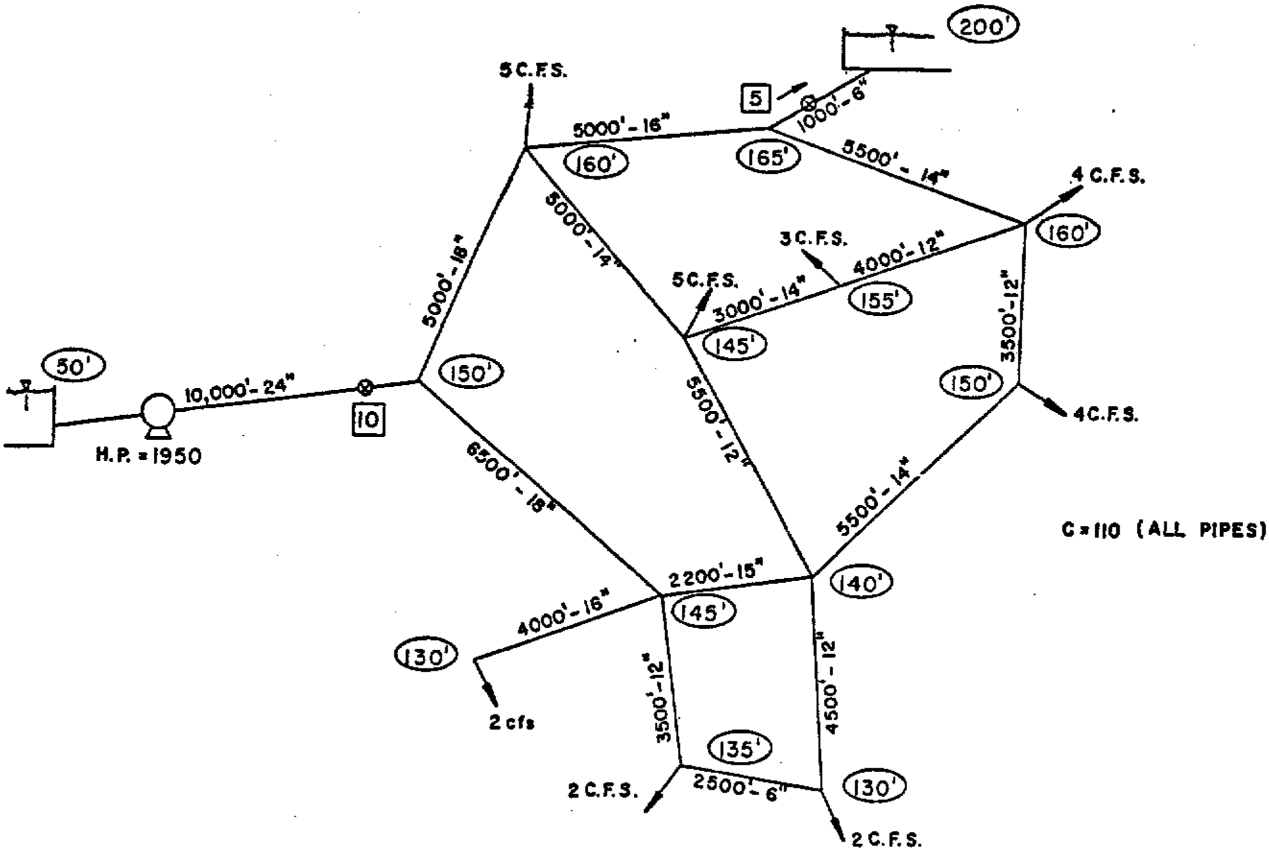
Example 2A Fourteen pipe distribution system changes for second analysis shown in ( ). English units p=14, ℓ=3, j=8, t=4



Hazen Williams Coefficient C=130 for all pipes

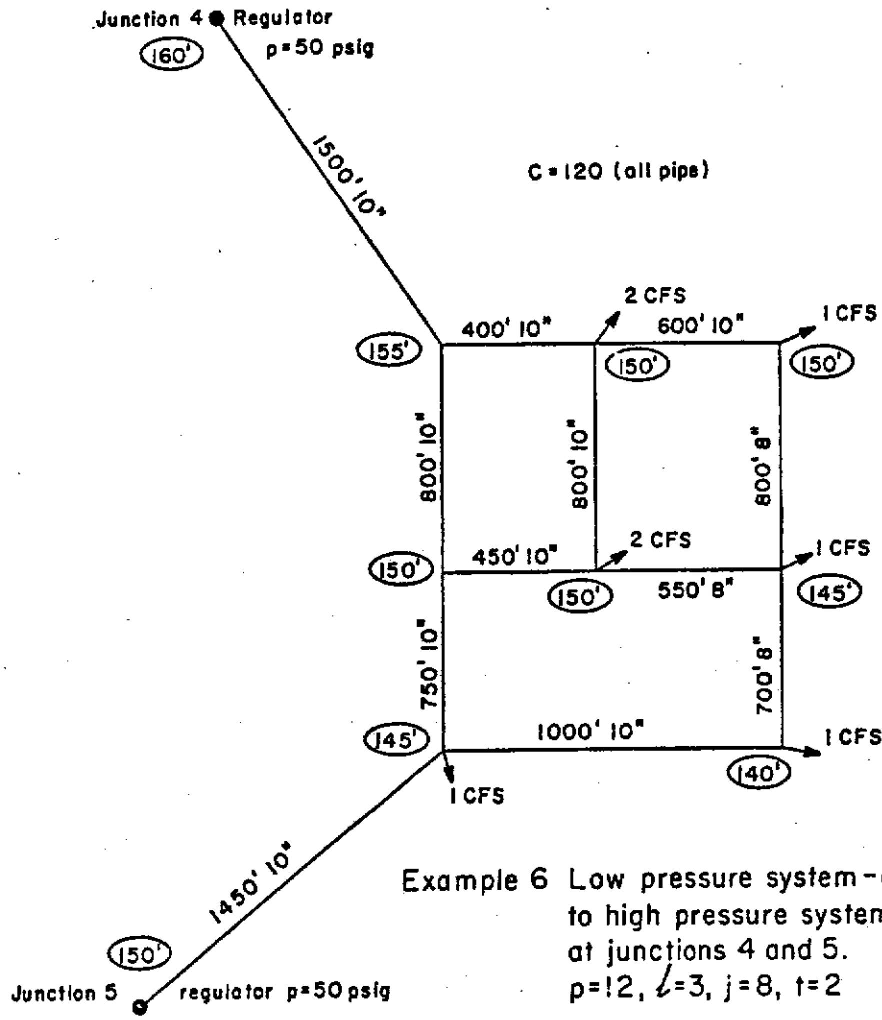
○ denotes sprinkler - M = 23

Example 4 17 pipe tree type watering system. p=17, ℓ=0, j=8, t=10



Example 5A 17 pipe water distribution system. p=17, ℓ=4, j=12, t=2

Schematics - Examples (cont.)



Example 6 Low pressure system-connected to high pressure system (example 5) at junctions 4 and 5.
$p=12$, $\ell=3$, $j=8$, $t=2$

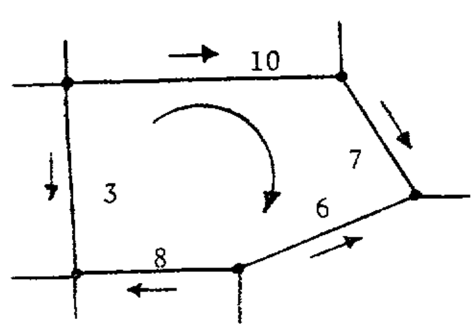
Schematics - Examples (cont.)

Computer Programs - Loop Equations

The BASIC programs written for algorithms based on the loop equations are very similar and use some additional common nomenclature. This is:

P2	- number of paths (ℓ + f - 1)
O(N)	- K_M (Equ. 8) for pipe N
C(N)	- K_p (Equ. 6) for pipe N
G2	- grade at a reference fixed grade node
E(N)	- energy difference for path N
G1	- G_i (gradient for pipe section based on Q_i, Equ. 19)
H1	- H_i (head change for pipe section based on Q_i, Equ. 18)
Q1	- absolute value of the flowrate
I	- absolute value of pipe number
T(N)	- head change for pipe section N
W(N, M)	- array for storing geometric data

The W(N, M) array stores the required geometric data for the network and the required data depends on the algorithm used. For all methods it is necessary to know the flow direction in each pipe section at all times because the required head summations depend on the flow direction in the pipe sections. This is done by designating an assumed flow direction in each pipe section. If the algorithm requires an initial flow assumption then the assumed flow direction is in the direction of the initial value used. If an assumed initial flowrate is not required an arbitrary direction is assumed. The initial assumed direction is indicated on the numbering schemes included with the computer results for each example. As the calculations are carried out, the direction of the flowrate is positive if it is in the initial assumed direction and negative if it reverses directions. Input geometric data requires a sign based on the initial assumed flow direction in order to specify the direction assumed. For example, all algorithms require input data specifying assigned pipe numbers for the paths of pipe sections making up primary loops and between fixed grade nodes. The pipe numbers are input with a postiive sign if the assumed flow direction is in the path direction taken to input the data and with a negative sign if it is not. This is illustrated using the loop shown below with a path direction (clockwise) as sepcified and the assumed flow directions noted. The data input would be 10,7,-6,8,-3. Additional sign convention for input geometric data are noted in the discussions and data specifications for the various algorithms.



APPENDIX 1 - Single Path Adjustment Method

The BASIC program uses the procedure described in pages 6-7 and the listing that follows. Additional pertinent notation used in the program is defined below with reference to the program line where it is first used.

270	Y(N)	- number of pipes in path between junction N and junction N-1
330	Y(N)	- number of pipes in path N
430	B1	- numerator for flow correction factor (Equ. 21)
440	A1	- denomenator for flow correction factor (Equ. 21)
530	Q2	- ΔQ (Equ. 21)

Note in line 510 the initial head changes, H_i, are added algebraicly and the correct sign is obtained by multiplying the head change by the flowrate and pipe number (with sign) and dividing by the absolute values of those quantities.

The data requirements are summarized below. An initial flowrate which satisfies continuity is required and the flow direction for these assumed flowrates are indicated on the example schematics. Pipe numbers are input for paths of pipes for the energy equation and paths of pipes between junction nodes and these are input with a positive sign if the initial flow direction is in the path direction and negative if it is opposite.

The required data is:

1 first line: no. of pipes, no. of junction nodes, flow conversion factor

2 next P lines (one for each pipe section): length (ft), diameter (in), roughness, Σ minor loss coefficients, useful pump power, initial flowrate

3 next line : reference grade for grade calculations, number of pipes from reference grade to junction node #1, pipe numbers (with sign) in path from reference grade to junction node #1 (repeat for each pipe), number of pipes between junction node #1 and #2, pipe numbers (with sign) in path from junction node #1 to #2 (repeat for each pipe), repeat last two items for each junction node in pipe system

4 next P-J lines (one for each energy equation). energy difference between starting and ending node (zero for loop), number of pipes in path, pipe numbers (with sign) in path from starting to ending node

The program listing and examples follow:

BASIC LISTING- SINGLE PATH ADJUSTMENT METHOD

```
100 DIM D(20),C(20),O(20),Z(20),V(20)
110 DIM E(20),Q(20),H(20),T(20),Y(20)
120 DIM W(20,20)
130 READ P,J,C1
140 P2=P-J
150 X=1.852
160 FOR N=1 TO P
170 READ L1,D1,R1,M1,P1,Q(N)
180 Q(N)=Q(N)/C1
190 D(N)=D1/12
200 O(N)=.02517*M1/D(N)**4
210 C(N)=4.73*L1/(R1**X*D(N)**4.87)
220 Z(N)=P1*550/62.4
230 V(N)=Q(N)
240 NEXT N
250 READ G2
260 FOR N=1 TO J
270 READ Y(N)
280 FOR M=1 TO Y(N)
290 READ W(N,M)
300 NEXT M
310 NEXT N
320 FOR N=J+1 TO P
330 READ E(N),Y(N)
340 FOR M=1 TO Y(N)
350 READ W(N,M)
360 NEXT M
370 NEXT N
380 PRINT"TRIAL NO.   RELATIVE FLOW CHANGE  AVERAGE FLOW CHANGE"
390 FOR T5=1 TO 100
400 Q8=0
410 Q9=0
420 FOR N=J+1 TO P
430 B1=E(N)
440 A1=0
450 FOR M=1 TO Y(N)
460 I=ABS(W(N,M))
470 Q1=ABS(Q(I))
480 G1=X*C(I)*Q1**(X-1)+2*O(I)*Q1+Z(I)/Q1**2
490 H1=C(I)*Q1**X+O(I)*Q1**2-Z(I)/Q1
500 A1=A1+G1
510 B1=B1-H1*W(N,M)*Q(I)/(Q1*I)
520 NEXT M
530 Q2=B1/A1
540 FOR M=1 TO Y(N)
550 I=ABS(W(N,M))
560 Q(I)=Q(I)+Q2*W(N,M)/I
570 NEXT M
580 NEXT N
590 FOR N=1 TO P
600 Q8=Q8+ABS(ABS(Q(N))-ABS(V(N)))
610 Q9=Q9+ABS(Q(N))
620 V(N)=Q(N)
630 NEXT N
640 Q5=Q8/Q9
650 Q8=Q8*C1/P
660 PRINT T5,Q5,Q8
670 IF Q5<.001 GO TO 690
680 NEXT T5
690 FOR N=1 TO P
700 Q1=ABS(Q(N))
710 T(N)=Q(N)*(C(N)*Q1**(X-1)+O(N)*Q1)-Z(N)/Q(N)
720 NEXT N
730 FOR N=1 TO J
740 FOR M=1 TO Y(N)
750 I=ABS(W(N,M))
760 G2=G2-T(I)*I/W(N,M)
770 NEXT M
780 H(N)=G2
790 NEXT N
800 PRINT
810 PRINT
820 PRINT"PIPE NUMBER       FLOWRATE"
830 FOR N=1 TO P
840 Q(N)=C1*ABS(Q(N))
850 PRINT N,Q(N)
860 NEXT N
870 PRINT
880 PRINT "NODE NUMBER          GRADE"
890 FOR N=1 TO J
900 PRINT N,H(N)
910 NEXT N
```

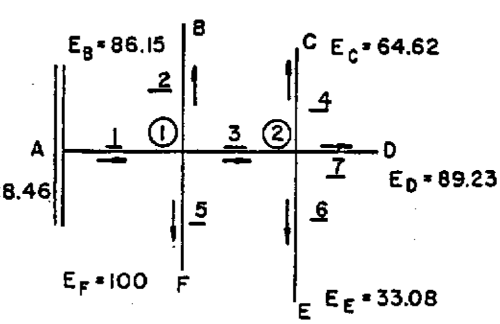
TRIAL NO.	RELATIVE FLOW CHANGE	AVERAGE FLOW CHANGE
1	.269477	63.3179
2	6.07418E-02	14.4689
3	1.75503E-02	4.18537
4	5.54139E-03	1.31961
5	1.47662E-03	.351439
6	3.21299E-04	7.64595E-02

PIPE NUMBER	FLOWRATE
1	608.556
2	95.59
3	448.674
4	82.193
5	64.2974
6	80.5535
7	285.928

NODE NUMBER	GRADE
1	130.434
2	97.4365

```
920 DATA 7,2,448.86
930 DATA 200,4,110,0,0,500
940 DATA 150,2,110,2,0,100
950 DATA 200,4,110,0,0,300
960 DATA 100,2,110,11,0,100
970 DATA 200,2,110,6,0,100
980 DATA 300,2,110,4,0,100
990 DATA 80,4,110,3,0,100
1000 DATA 188.46,1,1,1,3
1010 DATA 102.3,2,1,2
1020 DATA 123.84,3,1,3,4
1030 DATA 99.23,3,1,3,7
1040 DATA 155.38,3,1,3,6
1050 DATA 88.46,2,1,5
```

NUMBERING SCHEME



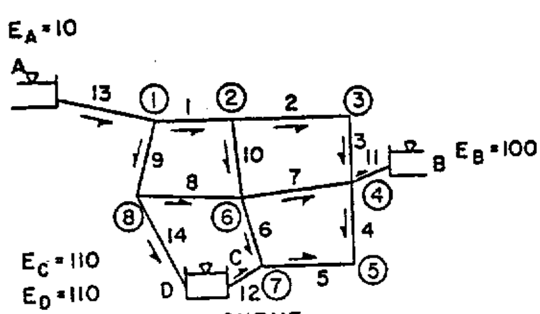
Example 1 - Single Path Adjustment Method

TRIAL NO.	RELATIVE FLOW CHANGE	AVERAGE FLOW CHANGE
1	7.82287E-02	.383143
2	3.36596E-02	.16466
3	4.10239E-03	2.00776E-02
4	1.00661E-03	4.92770E-03
5	4.61483E-04	2.25960E-03

PIPE NUMBER	FLOWRATE
1	9.65522
2	5.2743
3	1.2743
4	6.24794E-02
5	1.93752
6	3.36247
7	4.9054
8	3.88695
9	9.11653
10	4.38092
11	2.11722
12	.575045
13	18.7716
14	3.22958

NODE NUMBER	GRADE
1	202.543
2	133.575
3	104.774
4	103.046
5	103.04
6	119.833
7	106.58
8	136.198

```
1240 DATA 14,8,1
1250 DATA 2000,14,100,8,0,9
1260 DATA 3000,14,100,0,0,5
1270 DATA 2500,14,100,0,0,1
1280 DATA 2000,14,100,0,0,.5
1290 DATA 2000,14,100,10,0,1.5
1300 DATA 1500,12,100,0,0,3
1310 DATA 2000,14,100,0,0,5
1320 DATA 3000,14,100,0,0,4
1330 DATA 2500,14,100,0,0,9
1340 DATA 2000,14,100,0,0,4
1350 DATA 100,8,100,0,0,1.5
1360 DATA 200,6,100,10,0,.5
1370 DATA 5000,16,100,5,1000,18
1380 DATA 3200,12,100,0,0,3
1390 DATA 10,1,13,1,1,1,2,1,3,1,4,2,-4,-7,1,6,2,-6,-8
1400 DATA 0,4,1,10,-8,-9
1410 DATA 0,4,2,3,-7,-10
1420 DATA 0,4,7,4,-5,-6
1430 DATA -90,5,13,1,2,3,11
1440 DATA -100,3,13,9,14
1450 DATA -100,5,13,1,10,6,-12
```



NUMBERING SCHEME

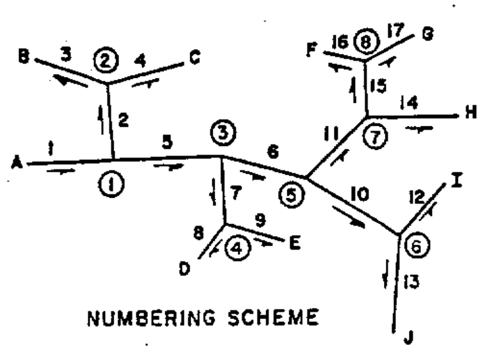
Example 2 - Single Path Adjustment Method

TRIAL NO.	RELATIVE FLOW CHANGE	AVERAGE FLOW CHANGE
1	6.26541E-02	10.6004
2	2.32765E-02	3.94101
3	1.29302E-02	2.18964
4	8.09564E-03	1.37038
5	4.88977E-03	.827152
6	2.87395E-03	.485851
7	1.80289E-03	.304653
8	1.23531E-03	.208695
9	9.16092E-04	.154755

PIPE NUMBER	FLOWRATE
1	635.671
2	105.634
3	52.4156
4	53.218
5	530.041
6	325.386
7	204.667
8	108.839
9	95.8279
10	160.563
11	164.824
12	87.8722
13	72.6905
14	55.4857
15	109.338
16	52.5974
17	56.7408

NODE NUMBER	GRADE
1	290.753
2	155.302
3	189.361
4	174.228
5	160.978
6	150.708
7	154.63
8	152.37

```
1460 DATA 17,8,448.86
1470 DATA 635,4,130,0,54.2,600
1480 DATA 520,2,130,5,0,100
1490 DATA 380,2,130,23,0,50
1500 DATA 510,2,130,23,0,50
1510 DATA 615,4,130,0,0,500
1520 DATA 425,4,130,0,0,300
1530 DATA 385,4,130,10,0,200
1540 DATA 195,2,130,23,0,100
1550 DATA 270,2,130,23,0,100
1560 DATA 540,4,130,2,0,150
1570 DATA 335,4,130,0,0,150
1580 DATA 210,2,130,23,0,75
1590 DATA 415,2,130,23,0,75
1600 DATA 375,2,130,23,0,50
1610 DATA 255,4,130,0,0,100
1620 DATA 180,2,130,23,0,50
1630 DATA 200,2,130,23,0,50
1640 DATA 100,1,1,1,2,2,-2,5,1,7,2,-7,6,1,10,2,-10,11,1,15
1650 DATA -20,3,1,2,3
1660 DATA -10,3,1,2,4
1670 DATA 20,4,1,5,7,8
1680 DATA 15,4,1,5,7,9
1690 DATA 20,5,1,5,6,10,13
1700 DATA 15,5,1,5,6,10,12
1710 DATA -15,5,1,5,6,11,14
1720 DATA -30,6,1,5,6,11,15,16
1730 DATA -25,6,1,5,6,11,15,17
```



NUMBERING SCHEME

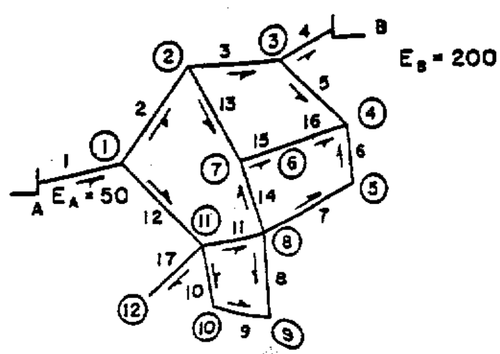
Example 4 - Single Path Adjustment Method

TRIAL NO.	RELATIVE FLOW CHANGE	AVERAGE FLOW CHANGE
1	.331088	2.36868
2	.141433	.907504
3	8.27044E-02	.508281
4	4.39672E-02	.266733
5	1.78693E-02	.108027
6	6.32577E-03	3.81919E-02
7	2.16323E-03	1.30548E-02
8	7.37755E-04	4.45161E-03

PIPE NUMBER	FLOWRATE
1	29.6052
2	16.4996
3	6.16067
4	2.60526
5	3.55541
6	.496099
7	4.4961
8	1.41175
9	.588243
10	2.58824
11	8.5173
12	13.1055
13	5.33903
14	2.60945
15	2.94848
16	5.15128E-02
17	2

NODE NUMBER	GRADE
1	474.483
2	376.676
3	348.679
4	327.359
5	328.108
6	327.346
7	335.568
8	361.037
9	354.357
10	375.804
11	391.769
12	388.98

```
1740 DATA 17,12,1
1750 DATA 10000,24,110,10,1950,40
1760 DATA 5000,18,110,0,0,25
1770 DATA 5000,16,110,0,0,15
1780 DATA 1000,6,110,5,0,13
1790 DATA 5500,14,110,0,0,2
1800 DATA 3500,12,110,0,0,1
1810 DATA 5500,14,110,0,0,5
1820 DATA 4500,12,110,0,0,1
1830 DATA 2500,6,110,0,0,1
1840 DATA 3500,12,110,0,0,3
1850 DATA 2200,15,110,0,0,10
1860 DATA 6500,18,110,0,0,15
1870 DATA 5000,14,110,0,0,5
1880 DATA 5500,12,110,0,0,4
1890 DATA 3000,14,110,0,0,4
1900 DATA 4000,12,110,0,0,1
1910 DATA 4000,16,110,0,0,2
1920 DATA 50,1,1,1,2,1,3,1,5,1,-6,2,6,-16
1930 DATA 1,-15,1,-14,1,8,1,-9,1,-10,1,17
1940 DATA 0,5,2,13,-14,-11,-12
1950 DATA 0,5,3,5,-16,-15,-13
1960 DATA 0,5,15,16,-6,-7,14
1970 DATA 0,4,11,8,-9,-10
1980 DATA -150 ,4,1,2,3,4
```



NUMBERING SCHEME

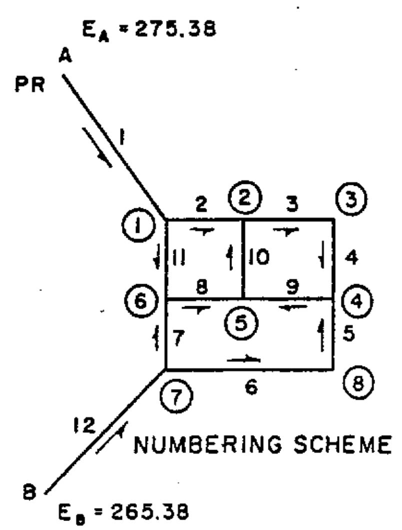
Example 5 - Single Path Adjustment Method

TRIAL NO.	RELATIVE FLOW CHANGE	AVERAGE FLOW CHANGE
1	.434211	.727235
2	.159398	.264427
3	4.35456E-02	7.13925E-02
4	1.25796E-02	2.05487E-02
5	2.82026E-03	4.60345E-03
6	5.78349E-04	9.43894E-04

PIPE NUMBER	FLOWRATE
1	4.29332
2	3.0762
3	.891887
4	.108113
5	.631613
6	1.63161
7	1.07506
8	2.29218
9	.476499
10	.184317
11	1.21711
12	3.70668

NODE NUMBER	GRADE
1	239.251
2	234.055
3	233.268
4	233.33
5	234.
6	237.39
7	238.78
8	234.766

```
1060 DATA 12,8,1
1070 DATA 1500,10,120,0,0,3
1080 DATA 400,10,120,0,0,2
1090 DATA 600,10,120,0,0,1.6
1100 DATA 800,8,120,0,0,.6
1110 DATA 700,8,120,0,0,1.5
1120 DATA 1000,10,120,0,0,2.5
1130 DATA 750,10,120,0,0,1.5
1140 DATA 450,10,120,0,0,2.5
1150 DATA 550,8,120,0,0,1.1
1160 DATA 800,10,120,0,0,1.6
1170 DATA 800,10,120,0,0,1
1180 DATA 1450,10,120,0,0,5
1190 DATA 275.38,1,1,1,2,1,3,1,4,1,9,1,-8,1,-7,1,6
1200 DATA 0,4,2,-10,-8,-11
1210 DATA 0,4,3,4,9,10
1220 DATA 0,5,8,-9,-5,-6,7
1230 DATA 10,4,1,11,-7,-12
```



Example 6 - Single Path Adjustment Method

APPENDIX 2 - Simultaneous Path Adjustment Method

The BASIC program which is listed on the next page uses the procedure described on pages 7-8. Additional pertinent notation used in the program is defined below along with reference to the program number where is it first used.

170	MAT A	- coefficient matrix for flow adjustment equations
180	MAT R	- inverse of A
190	MAT B	- constant matrix (vector) for right side of flow adjustment equations
200	MAT G	- solutions of flow adjustment equations (ΔQ's)
290	V(N)	- initial flowrate for pipe section N(Q_i)
330	Y(N)	- number of pipes in path between junction node N and N-1
390	Y(N)	- number of pipes in path N
390	U(N)	- number of pipes common to other paths
510	N1	- path number
590	A1	- ΣG_i for path N1 (also A(N1, N1))
600	B1	- $\Delta E - \Sigma H_i$ for path N1 (also B(N1, 1))
670	I1	- path number for common pipe
680	I2	- pipe number for common pipe
720	A(N1, I1)	- contribution from path I1 to flow adjustment for path N1

Note in lines 600 and 720 contributions are added algebraically depending on assumed flow directions and path directions.

The data requirements are summarized below. An initial flowrate which satisfies continuity is required and the flow direction for these assumed flowrates are indicated on the example schematics. Pipe numbers are input for paths of pipes for the energy equation and paths of pipes between junction nodes and these are input with a positive sign if the initial flow direction is in the path direction and negative if it is opposite. For each path, path numbers (with a sign) and pipe numbers are input for all pipes common to other paths and the path number is input with a positive sign if the two paths are in the same direction with respect to the common pipe.

The required data is:

1. first line: no. of pipes, no. of junction nodes, flow conversion factor
2. next P lines (one for each pipe section): length (ft), diameter (in), roughness, Σ minor loss coefficients, useful pump power, initial flowrate
3. next line : reference grade for grade claculations, number of pipes from reference grade to junction node #1, pipe numbers

(with sign) in path from reference grade to junction node #1 (repeat for each pipe), number of pipes between junction node #1 and #2, pipe numbers (with sign) in path from junction node #1 to #2 (repeat for each pipe), repeat last two items for each junction node in pipe system

4. next P-J lines (one for each energy equation): energy difference between starting and ending node (zero for loop), number of pipes in path, number of pipes common to other paths, pipe numbers (with sign) in path from starting to ending node, path number (with sign) for common pipe, pipe number for common pipe (repeat last two items for all common pipes in path).

A program listing and examples follow.

BASIC LISTING - SIMULTANEOUS PATH ADJUSTMENT METHOD

```
100 DIM A(20,20),R(20,20),B(20,1),G(20,1)
110 DIM D(20),C(20),O(20),Z(20),V(20)
120 DIM U(20)
130 DIM E(20),Q(20),H(20),T(20),Y(20)
140 DIM W(20,20)
150 READ P,J,C1
160 P2=P-J
170 MAT A =ZER(P2,P2)
180 MAT R =ZER(P2,P2)
190 MAT B =ZER(P2,1)
200 MAT G =ZER(P2,1)
210 X=1.852
220 FOR N=1 TO P
230 READ L1,D1,R1,M1,P1,Q(N)
240 Q(N)=Q(N)/C1
250 D(N)=D1/12
260 O(N)=.02517*M1/D(N)**4
270 C(N)=4.73*L1/(R1**X*D(N)**4.87)
280 Z(N)=P1*550/62.4
290 V(N)=Q(N)
300 NEXT N
310 READ G2
320 FOR N=1 TO J
330 READ Y(N)
340 FOR M=1 TO Y(N)
350 READ W(N,M)
360 NEXT M
370 NEXT N
380 FOR N=J+1 TO P
390 READ E(N),Y(N),U(N)
400 N1=Y(N)+2*U(N)
410 FOR M=1 TO N1
420 READ W(N,M)
430 NEXT M
440 NEXT N
450 PRINT'TRIAL NO.   RELATIVE FLOW CHANGE  AVERAGE FLOW CHANGE'
460 FOR T5=1 TO 20
470 MAT A =ZER(P2,P2)
480 Q8=0
490 Q9=0
```

```
500 FOR N=J+1 TO P
510 N1=N-J
520 B1=E(N)
530 A1=0
540 FOR M=1 TO Y(N)
550 I=ABS(W(N,M))
560 Q1=ABS(Q(I))
570 G1=X*C(I)*Q1**(X-1)+2*O(I)*Q1+Z(I)/Q1**2
580 H1=C(I)*Q1**X+O(I)*Q1**2-Z(I)/Q1
590 A1=A1+G1
600 B1=B1-H1*W(N,M)*Q(I)/(Q1*I)
610 NEXT M
620 A(N1,N1)=A1
630 B(N1,1)=B1
640 N2=Y(N)+1
650 N3=Y(N)+2*U(N)
660 FOR M=N2 TO N3 STEP 2
670 I1=W(N,M)
680 I2=W(N,M+1)
690 I=ABS(I2)
700 Q1=ABS(Q(I))
710 G1=X*C(I)*Q1**(X-1)+2*O(I)*Q1+Z(I)/Q1**2
720 A(N1,I1)=A(N1,I1)+G1*I2/I
730 NEXT M
740 NEXT N
750 MAT R=INV(A)
760 MAT G=R*B
770 FOR N=J+1 TO P
780 N1=N-J
790 FOR M=1 TO Y(N)
800 I=ABS(W(N,M))
810 Q(I)=Q(I)+G(N1,1)*W(N,M)/I
820 NEXT M
830 NEXT N
840 FOR N=1 TO P
850 Q8=Q8+ABS(ABS(Q(N))-ABS(V(N)))
860 Q9=Q9+ABS(Q(N))
870 V(N)=Q(N)
880 NEXT N
890 Q5=Q8/Q9
900 Q8=Q8*C1/P
910 PRINT T5,Q5,Q8
920 IF Q5<.001 GO TO 940
930 NEXT T5
940 FOR N=1 TO P
950 Q1=ABS(Q(N))
960 T(N)=Q(N)*(C(N)*Q1**(X-1)+O(N)*Q1)-Z(N)/Q(N)
970 NEXT N
980 FOR N=1 TO J
990 FOR M=1 TO Y(N)
1000 I=ABS(W(N,M))
1010 G2=G2-T(I)*I/W(N,M)
1020 NEXT M
1030 H(N)=G2
1040 NEXT N
1050 PRINT
1060 PRINT
1070 PRINT"PIPE NUMBER      FLOWRATE"
1080 FOR N=1 TO P
1090 Q(N)=C1*ABS(Q(N))
1100 PRINT N,Q(N)
1110 NEXT N
1120 PRINT
1130 PRINT "NODE NUMBER          GRADE"
1140 FOR N=1 TO J
1150 PRINT N,H(N)
1160 NEXT N
```

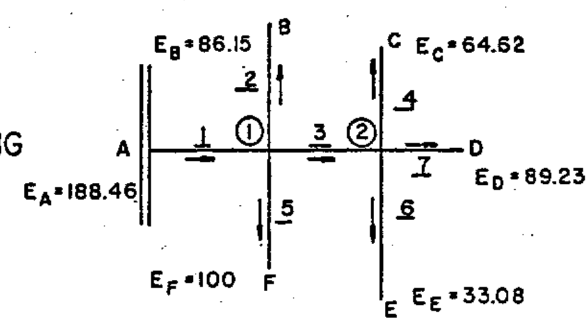
TRIAL NO.	RELATIVE FLOW CHANGE	AVERAGE FLOW CHANGE
1	.346501	86.663
2	5.49169E-02	13.0842
3	1.33345E-03	.317296
4	1.28497E-06	3.05762E-04

PIPE NUMBER	FLOWRATE
1	608.524
2	95.6187
3	448.614
4	82.1591
5	64.2924
6	80.5549
7	285.9

NODE NUMBER	GRADE
1	130.439
2	97.4503

```
1170 DATA 7,2,448.86
1180 DATA 200,4,110,0,0,500
1190 DATA 150,2,110,2,0,100
1200 DATA 200,4,110,0,0,300
1210 DATA 100,2,110,11,0,100
1220 DATA 200,2,110,6,0,100
1230 DATA 300,2,110,4,0,100
1240 DATA 80,4,110,3,0,100
1250 DATA 188.46,1,1,1,3
1260 DATA 102.3,2,1,1,2,2,-2
1270 DATA -13.85,2,2,-2,5,1,-2,3,-5
1280 DATA 35.38,3,2,-5,3,4,2,-5,4,-4
1290 DATA 31.54,2,2,-4,6,3,-4,5,-6
1300 DATA -56.15,2,1,-6,7,4,-6
```

NUMBERING
SCHEME



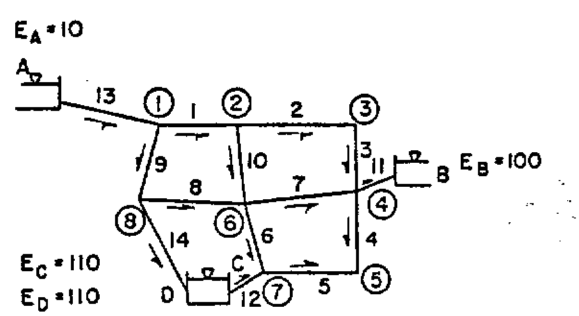
Example 1 - Simultaneous Path Adjustment Method

TRIAL NO.	RELATIVE FLOW CHANGE	AVERAGE FLOW CHANGE
1	6.58459E-02	.322297
2	4.26452E-03	2.08892E-02
3	1.76294E-03	8.63916E-03
4	9.91450E-04	4.85968E-03

PIPE NUMBER	FLOWRATE
1	9.65679
2	5.27502
3	1.27502
4	5.42148E-02
5	1.94578
6	3.35285
7	4.9259
8	3.89698
9	9.11953
10	4.38177
11	2.14671
12	.592932
13	18.7762
14	3.22254

NODE NUMBER	GRADE
1	202.301
2	133.312
3	104.504
4	102.773
5	102.769
6	119.691
7	106.508
8	136.134

```
1490 DATA 14,8,1
1500 DATA 2000,14,100,8,0,9
1510 DATA 3000,14,100,0,0,5
1520 DATA 2500,14,100,0,0,1
1530 DATA 2000,14,100,0,0,.5
1540 DATA 2000,14,100,10,0,1.5
1550 DATA 1500,12,100,0,0,3
1560 DATA 2000,14,100,0,0,5
1570 DATA 3000,14,100,0,0,4
1580 DATA 2500,14,100,0,0,9
1590 DATA 2000,14,100,0,0,4
1600 DATA 100,8,100,0,0,1.5
1610 DATA 200,6,100,10,0,.5
1620 DATA 5000,16,100,5,1000,18
1630 DATA 3200,12,100,0,0,3
1640 DATA 10,1,13,1,1,1,2,1,3,1,4,2,-4,-7,1,6,2,-6,-8
1650 DATA 0,4,5,1,10,-8,-9,2,-10,4,1,5,-9,6,1,6,10
1660 DATA 0,4,5,2,3,-7,-10,1,-10,3,-7,4,2,4,3,6,-10
1670 DATA 0,4,2,7,4,-5,-6,2,-7,6,-6
1680 DATA -90,5,6,13,1,2,3,11,1,1,2,2,2,3,5,13,6,1,6,13
1690 DATA -100,3,3,13,9,14,1,-9,4,13,6,13
1700 DATA -100,5,7,13,1,10,6,-12,1,1,1,10,2,-10,3,-6,4,1,4,13,5,13
```



NUMBERING SCHEME

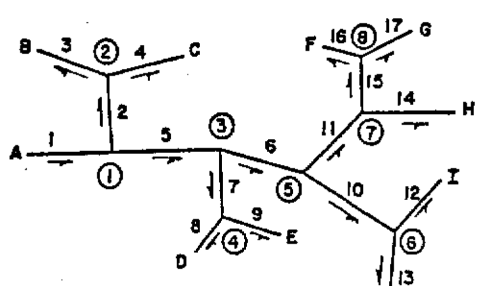
Example 2 - Simultaneous Path Adjustment Method

TRIAL NO.	RELATIVE FLOW CHANGE	AVERAGE FLOW CHANGE
1	5.74509E-02	9.64187
2	1.06125E-02	1.79142
3	1.53770E-03	.259765
4	2.06060E-04	3.48134E-02

PIPE NUMBER	FLOWRATE
1	635.783
2	105.559
3	52.3444
4	53.2148
5	530.223
6	325.615
7	204.609
8	108.746
9	95.8634
10	160.982
11	164.633
12	88.1425
13	72.8398
14	55.699
15	108.934
16	52.4047
17	56.529

NODE NUMBER	GRADE
1	290.645
2	155.372
3	189.189
4	174.063
5	160.769
6	150.448
7	154.434
8	152.189

```
1710 DATA 17,8,448.86
1720 DATA 635,4,130,0,54.2,600
1730 DATA 520,2,130,5,0,100
1740 DATA 380,2,130,23,0,50
1750 DATA 510,2,130,23,0,50
1760 DATA 615,4,130,0,0,500
1770 DATA 425,4,130,0,0,300
1780 DATA 385,4,130,10,0,200
1790 DATA 195,2,130,23,0,100
1800 DATA 270,2,130,23,0,100
1810 DATA 540,4,130,2,0,150
1820 DATA 335,4,130,0,0,150
1830 DATA 210,2,130,23,0,75
1840 DATA 415,2,130,23,0,75
1850 DATA 375,2,130,23,0,50
1860 DATA 255,4,130,0,0,100
1870 DATA 180,2,130,23,0,50
1880 DATA 200,2,130,23,0,50
1890 DATA 100,1,1,1,2,2,-2,5,1,7,2,-7,6,1,10,2,-10,11,1,15
1900 DATA -20,3,2,1,2,3,3,-2,2,-3
1910 DATA 10,2,2,-3,4,1,-3,3,-4
1920 DATA 30,5,4,-4,-2,5,7,8,2,-4,1,-2,5,-7,4,-8
1930 DATA -5,2,2,-8,9,3,-8,5,-9
1940 DATA 5,5,3,-9,-7,6,10,13,4,-9,7,-10,6,-13
1950 DATA -5,2,2,-13,12,5,-13,7,-12
1960 DATA -30,4,3,-12,-10,11,14,6,-12,8,-14,5,-10
1970 DATA -10,3,2,-14,15,17,7,-14,9,-17
1980 DATA -5,2,1,-17,16,8,-17
```



NUMBERING SCHEME

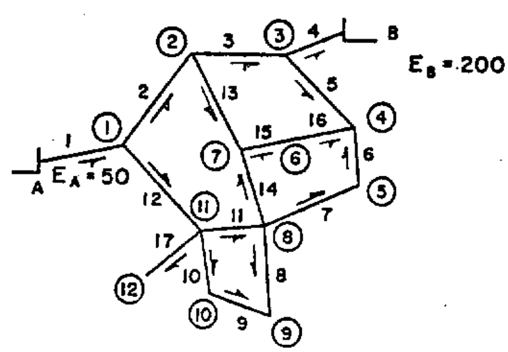
Example 4 - Simultaneous Path Adjustment Method

TRIAL NO.	RELATIVE FLOW CHANGE	AVERAGE FLOW CHANGE
1	.274315	1.89649
2	.125239	.778823
3	3.26663E-02	.197532
4	2.61657E-03	1.57874E-02
5	1.67876E-05	1.01288E-04

PIPE NUMBER	FLOWRATE
1	29.6046
2	16.5049
3	6.16393
4	2.60463
5	3.5593
6	.49289
7	4.49289
8	1.41217
9	.587826
10	2.58782
11	8.51183
12	13.0997
13	5.34104
14	2.60676
15	2.94781
16	5.21863E-0?
17	2

NODE NUMBER	GRADE
1	474.502
2	376.637
3	348.613
4	327.25
5	327.99
6	327.236
7	335.455
8	360.875
9	354.191
10	375.611
11	391.57
12	388.782

```
1990 DATA 17,12,1
2000 DATA 10000,24,110,10,1950,40
2010 DATA 5000,18,110,0,0,25
2020 DATA 5000,16,110,0,0,15
2030 DATA 1000,6,110,5,0,13
2040 DATA 5500,14,110,0,0,2
2050 DATA 3500,12,110,0,0,1
2060 DATA 5500,14,110,0,0,5
2070 DATA 4500,12,110,0,0,1
2080 DATA 2500,6,110,0,0,1
2090 DATA 3500,12,110,0,0,3
2100 DATA 2200,15,110,0,0,10
2110 DATA 6500,18,110,0,0,15
2120 DATA 5000,14,110,0,0,5
2130 DATA 5500,12,110,0,0,4
2140 DATA 3000,14,110,0,0,4
2150 DATA 4000,12,110,0,0,1
2160 DATA 4000,16,110,0,0,2
2170 DATA 50,1,1,1,2,1,3,1,5,1,-6,2,6,-16
2180 DATA 1,-15,1,-14,1,8,1,-9,1,-10,1,17
2190 DATA 0,5,4,2,13,-14,-11,-12,5,2,2,-13,3,-14,4,-11
2200 DATA 0,5,4,3,5,-16,-15,-13,5,3,3,-16,3,-15,1,-13
2210 DATA 0,5,3,15,16,-6,-7,14,2,-15,2,-16,1,-14
2220 DATA 0,4,1,11,8,-9,-10,1,-11
2230 DATA -150,4,2,1,2,3,4,1,2,2,3
```



NUMBERING SCHEME

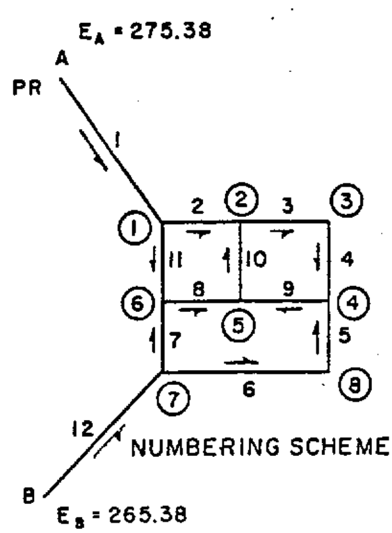
Example 5 - Simultaneous Path Adjustment Method

TRIAL NO.	RELATIVE FLOW CHANGE	AVERAGE FLOW CHANGE
1	.401845	.690139
2	.125886	.203871
3	4.35776E-02	7.12019E-02
4	3.94722E-03	6.44193E-03
5	6.10902E-05	9.96987E-05

PIPE NUMBER	FLOWRATE
1	4.29337
2	3.07668
3	.892329
4	.107671
5	.631427
6	1.63142
7	1.0752
8	2.29189
9	.476243
10	.184349
11	1.21669
12	3.70662

NODE NUMBER	GRADE
1	239.25
2	234.052
3	233.265
4	233.327
5	233.996
6	237.385
7	238.776
8	234.763

```
1310 DATA 12,8,1
1320 DATA 1500,10,120,0,0,3
1330 DATA 400,10,120,0,0,2
1340 DATA 600,10,120,0,0,1.6
1350 DATA 800,8,120,0,0,.6
1360 DATA 700,8,120,0,0,1.5
1370 DATA 1000,10,120,0,0,2.5
1380 DATA 750,10,120,0,0,1.5
1390 DATA 450,10,120,0,0,2.5
1400 DATA 550,8,120,0,0,1.1
1410 DATA 800,10,120,0,0,1.6
1420 DATA 800,10,120,0,0,1
1430 DATA 1450,10,120,0,0,5
1440 DATA 275.38,1,1,1,2,1,3,1,4,1,9,1,-8,1,-7,1,6
1450 DATA 0,4,3,2,-10,-8,-11,2,-10,3,-8,4,-11
1460 DATA 0,4,2,3,4,9,10,1,-10,3,-9
1470 DATA 0,5,2,8,-9,-5,-6,7,1,-8,2,-9
1480 DATA 10,4,2,1,11,-7,-12,1,-11,3,-7
```



Example 6 - Simultaneous Path Adjustment Method

APPENDIX 3 - Linear Method

This program is based on the procedure given on page 8. The listing of the BASIC program for this is shown on the next page. Additional pertinent notation along with the line number where it first appears is defined below.

150	MAT A	- coefficient matrix for linear equations
160	MAT R	- inverse of A
170	MAT B	- constants for linear equations
180	MAT G	- solution for flowrates (Q)
300	N1	- number of pipes connecting at junction
300	F1	- external demand or inflow at junction
330	P1	- pipe number (with sign) connecting junction
370	Y(N)	- number of pipes connecting junction N to junction N-1
430	Y(N)	- number of pipes in path N
530	B1	- $\Sigma(G_i Q_i - H_i)$ Equ. 21

The data requirements are summarized below. Pipe numbers are input for pipes connecting junction nodes, pipes in a path between junction nodes and pipes in a path for the energy equations. An assumed flow direction is made for each pipe section (as noted on the examples) and the following sign convention is applied. For pipes connecting a junction node the sign of the pipe number is input as negative for the assumed flow direction into the junction node and positive if it is out. For pipes in a path the sign of the pipe number is input as positive if it is in the path direction and negative if it is opposite.

The required data is:

1 first line: no. of pipes, no. of junction nodes, flow conversion factor.

2 next P lines (one for each pipe section): length (ft), diameter (in), roughness, Σ minor loss coefficients, useful pump power

3 one line: grade for reference

4 next J lines (one for each junction node): no. of pipes connecting node, external flowrate, pipe numbers (with sign) of pipes connecting this junction node, number of pipes in path connecting this junction node with last one input (reference grade for first junction node input), pipe numbers (with sign) of pipes in this path

5 next P-J lines (one for each energy equation): energy difference between starting and ending node (zero for loop), no. of pipes in path, pipe numbers (with sign) in path from starting to ending node

The program listing and examples follow.

```
100 DIM A(20,20),R(20,20),B(20,1),G(20,1)
110 DIM D(20),C(20),O(20),Z(20)
120 DIM E(20),Q(20),H(20),T(20),Y(20)
130 DIM W(20,10)
140 READ P,J,C1
150 MAT A =ZER(P,P)
160 MAT R =ZER(P,P)
170 MAT B =ZER(P,1)
180 MAT G =ZER(P,1)
190 X=1.852
200 FOR N=1 TO P
210 READ L1,D1,R1,M1,P1
220 D(N)=D1/12
230 O(N)=.02517*M1/D(N)**4
240 C(N)=4.73*L1/(R1**X*D(N)**4.87)
250 Z(N)=P1*550/62.4
260 Q(N)=3.1416*D(N)*D(N)
270 NEXT N
280 READ G2
290 FOR N=1 TO J
300 READ N1,F1
310 B(N,1)=-F1/C1
320 FOR M=1 TO N1
330 READ P1
340 I=ABS(P1)
350 A(N,I)=P1/I
360 NEXT M
370 READ Y(N)
380 FOR M=1 TO Y(N)
390 READ W(N,M)
400 NEXT M
410 NEXT N
420 FOR N=J+1 TO P
430 READ E(N),Y(N)
440 FOR M=1 TO Y(N)
450 READ W(N,M)
460 NEXT M
470 NEXT N
480 PRINT"TRIAL NO.   RELATIVE FLOW CHANGE  AVERAGE FLOW CHANGE"
490 FOR T5=1 TO 20
500 Q8=0
510 Q9=0
520 FOR N=J+1 TO P
530 B1=0
540 FOR M=1 TO Y(N)
550 I=ABS(W(N,M))
560 Q1=ABS(Q(I))
570 G1=X*C(I)*Q1**(X-1)+2*O(I)*Q1+Z(I)/Q1**2
580 H1=C(I)*Q1**X+O(I)*Q1**2-Z(I)/Q1
590 A(N,I)=G1*W(N,M)/I
600 B1=B1+(G1*Q1-H1)*Q(I)*W(N,M)/(Q1*I)
610 NEXT M
620 B(N,1)=B1+E(N)
630 NEXT N
640 MAT R=INV(A)
650 MAT G=R*B
660 FOR N=1 TO P
670 Q8=Q8+ABS(ABS(Q(N))-ABS(G(N,1)))
680 Q9=Q9+ABS(G(N,1))
690 Q(N)=G(N,1)
700 NEXT N
710 Q5=Q8/Q9
720 Q8=Q8*C1/P
730 PRINT T5,Q5,Q8
740 IF Q5<.001 GO TO 760
750 NEXT T5
760 FOR N=1 TO P
770 Q1=ABS(Q(N))
780 T(N)=Q(N)*(C(N)*Q1**(X-1)+O(N)*Q1)-Z(N)/Q(N)
790 NEXT N
800 FOR N=1 TO J
810 FOR M=1 TO Y(N)
820 I=ABS(W(N,M))
830 G2=G2-T(I)*W(N,M)/I
840 NEXT M
850 H(N)=G2
860 NEXT N
870 PRINT
880 PRINT
890 PRINT"PIPE NUMBER      FLOWRATE"
900 FOR N=1 TO P
910 Q(N)=C1*ABS(Q(N))
920 PRINT N,Q(N)
930 NEXT N
940 PRINT
950 PRINT "NODE NUMBER          GRADE"
960 FOR N=1 TO J
970 PRINT N,H(N)
980 NEXT N
```

BASIC LISTING - LINEAR METHOD

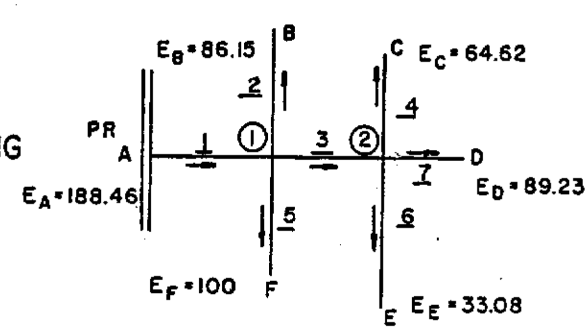
TRIAL NO.	RELATIVE FLOW CHANGE	AVERAGE FLOW CHANGE
1	.763143	288.47
2	.449275	117.181
3	9.19791E-02	21.9694
4	3.72790E-03	.88711
5	7.01881E-06	1.67022E-03

PIPE NUMBER	FLOWRATE
1	608.566
2	95.61
3	448.616
4	82.1092
5	64.3402
6	80.5695
7	285.936

NODE NUMBER	GRADE
1	130.432
2	97.4426

```
1010 DATA 7,2,448.86
1020 DATA 200,4,110,0,0
1030 DATA 150,2,110,2,0
1040 DATA200,4,110,0,0
1050 DATA 100,2,110,11,0
1060 DATA 200,2,110,6,0
1070 DATA 300,2,110,4,0
1080 DATA 80,4,110,3,0
1090 DATA 188.46
1100 DATA 4,0,-1,2,3,5,1,1
1110 DATA 4,0,-3,4,6,7,1,3
1120 DATA 102.3,2,1,2
1130 DATA 123.8,3,1,3,4
1140 DATA 99.23,3,1,3,7
1150 DATA 155.4,3,1,3,6
1160 DATA 88.5,2,1,5
```

NUMBERING SCHEME



Example 1 - Linear Method

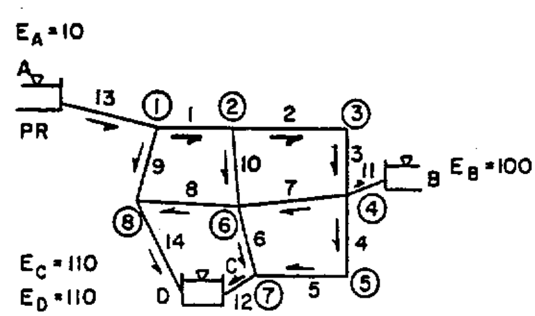
TRIAL NO.	RELATIVE FLOW CHANGE	AVERAGE FLOW CHANGE
1	.886948	2.52863
2	.554163	2.37176
3	.285895	1.41931
4	1.86452E-02	9.14687E-02
5	1.21549E-03	5.95961E-03
6	1.35507E-05	6.64395E-05

PIPE NUMBER	FLOWRATE
1	9.65666
2	5.27246
3	1.27246
4	3.44235E-02
5	1.96557
6	3.36371
7	4.91862
8	3.89813
9	9.11977
10	4.3842
11	2.15665
12	.601854
13	18.7764
14	3.22164

NODE NUMBER	GRADE
1	202.294
2	133.306
3	104.525
4	102.8
5	102.799
6	119.672
7	106.41
8	136.124

TIME 0 SECS.

```
1500 DATA 14,8,1
1510 DATA 2000,14,100,8,0
1520 DATA 3000,14,100,0,0
1530 DATA 2500,14,100,0,0
1540 DATA 2000,14,100,0,0
1550 DATA 2000,14,100,10,0
1560 DATA 1500,12,100,0,0
1570 DATA 2000,14,100,0,0
1580 DATA 3000,14,100,0,0
1590 DATA 2500,14,100,0,0
1600 DATA 2000,14,100,0,0
1610 DATA 100,8,100,0,0
1620 DATA  200,6,100,10,0
1630 DATA 5000,16,100,5,1000
1640 DATA 3200,12,100,0,0
1650 DATA 10
1660 DATA 3,0,1,9,-13,1,13
1670 DATA 3,0,-1,2,10,1,1
1680 DATA 2,4,-2,3,1,2
1690 DATA 4,4,-3,4,7,11,1,3
1700 DATA 2,2,-4,5,1,4
1710 DATA 4,0,6,-7,8,-10,2,5,-6
1720 DATA 3,2,-5,-6,12,1,6
1730 DATA 3,2,-8,-9,14,2,-6,8
1740 DATA 0,4,1,10,8,-9
1750 DATA 0,4,2,3,7,-10
1760 DATA 0,4,-7,4,5,-6
1770 DATA -90,5,13,1,2,3,11
1780 DATA -10,4,-11,4,5,12
1790 DATA 0,4,-14,-8,6,12
```



NUMBERING SCHEME

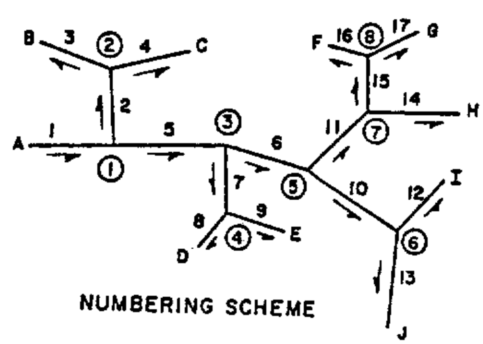
Example 2 - Linear Method

TRIAL NO.	RELATIVE FLOW CHANGE	AVERAGE FLOW CHANGE
1	.619864	48.3676
2	.439292	61.1331
3	.171004	28.6778
4	7.40561E-03	1.2512
5	5.53590E-05	9.35293E-03

PIPE NUMBER	FLOWRATE
1	635.787
2	105.558
3	52.3438
4	53.2143
5	530.229
6	325.627
7	204.603
8	108.743
9	95.8601
10	160.979
11	164.648
12	88.1411
13	72.8383
14	55.7034
15	108.944
16	52.4102
17	56.5344

NODE NUMBER	GRADE
1	290.642
2	155.372
3	189.183
4	174.059
5	160.762
6	150.441
7	154.426
8	152.181

```
1800 DATA 17,8,448.86
1810 DATA 635,4,130,0,54.2
1820 DATA 520,2,130,5,0
1830 DATA 380,2,130,23,0
1840 DATA 510,2,130,23,0
1850 DATA 615,4,130,0,0
1860 DATA 425,4,130,0,0
1870 DATA 385,4,130,10,0
1880 DATA 195,2,130,23,0
1890 DATA 270,2,130,23,0
1900 DATA 540,4,130,2,0
1910 DATA 335,4,130,0,0
1920 DATA 210,2,130,23,0
1930 DATA 415,2,130,23,0
1940 DATA 375,2,130,23,0
1950 DATA 255,4,130,0,0
1960 DATA 180,2,130,23,0
1970 DATA 200,2,130,23,0
1975 DATA 100
1980 DATA 3,0,-1,2,5,1,1
1990 DATA 3,0,-2,3,4,1,2
2000 DATA 3,0,-5,6,7,2,-2,5
2010 DATA 3,0,-7,8,9,1,7
2020 DATA 3,0,-6,10,11,2,-7,6
2030 DATA 3,0,-10,12,13,1,10
2040 DATA 3,0,-11,14,15,2,-10,11
2050 DATA 3,0,-15,16,17,1,15
2060 DATA -20,3,1,2,3
2070 DATA -10,3,1,2,4
2080 DATA -30,6,1,5,6,11,15,16
2090 DATA -25,6,1,5,6,11,15,17
2100 DATA -15,5,1,5,6,11,14
2110 DATA 15,5,1,5,6,10,12
2120 DATA 20,5,1,5,6,10,13
2130 DATA 15,4,1,5,7,9
2140 DATA 20,4,1,5,7,8
```



NUMBERING SCHEME

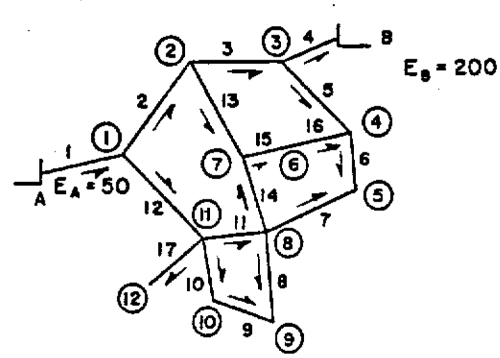
Example 4 - Linear Method

TRIAL NO.	RELATIVE FLOW CHANGE	AVERAGE FLOW CHANGE
1	.52431	2.6705
2	.200505	1.10147
3	.176449	1.13216
4	6.33941E-02	.385538
5	9.16941E-03	5.53335E-02
6	2.02230E-04	1.22016E-03

PIPE NUMBER	FLOWRATE
1	29.6046
2	16.5049
3	6.16392
4	2.60463
5	3.55929
6	.492892
7	4.49289
8	1.41217
9	.587825
10	2.58783
11	8.51182
12	13.0997
13	5.34103
14	2.60676
15	2.9478
16	5.21917E-02
17	2

NODE NUMBER	GRADE
1	474.503
2	376.638
3	348.613
4	327.25
5	327.99
6	327.237
7	335.455
8	360.875
9	354.192
10	375.611
11	391.571
12	388.782

```
2150 DATA 17,12,1
2160 DATA 10000,24,110,10,1950
2170 DATA 5000,18,110,0,0
2180 DATA 5000,16,110,0,0
2190 DATA 1000,6,110,5,0
2200 DATA 5500,14,110,0,0
2210 DATA 3500,12,110,0,0
2220 DATA 5500,14,110,0,0
2230 DATA 4500,12,110,0,0
2240 DATA 2500,6,110,0,0
2250 DATA 3500,12,110,0,0
2260 DATA 2200,15,110,0,0
2270 DATA 6500,18,110,0,0
2280 DATA 5000,14,110,0,0
2290 DATA 5500,12,110,0,0
2300 DATA 3000,14,110,0,0
2310 DATA 4000,12,110,0,0
2320 DATA 4000,16,110,0,0
2325 DATA 50
2330 DATA 3,0,-1,2,12,1,1
2340 DATA 3,5,-2,3,13,1,2
2350 DATA 3,0,-3,4,5,1,3
2360 DATA 3,4,-5,6,-16,1,5
2370 DATA 2,4,-6,-7,1,6
2380 DATA 2,3,-15,16,2,-6,-16
2390 DATA 3,5,-13,-14,15,1,-15
2400 DATA 4,0,7,8,-11,14,1,-14
2410 DATA 2,2,-8,-9,1,8
2420 DATA 2,2,9,-10,1,-9
2430 DATA 4,0,10,11,-12,17,1,-10
2440 DATA 1,2,-17,1,17
2450 DATA 0,5,2,13,-14,-11,-12
2460 DATA 0,5,3,5,-16,-15,-13
2470 DATA 0,5,14,15,16,6,-7
2480 DATA 0,4,11,8,-9,-10
2490 DATA 150,4,-4,-3,-2,-1
```



NUMBERING SCHEME

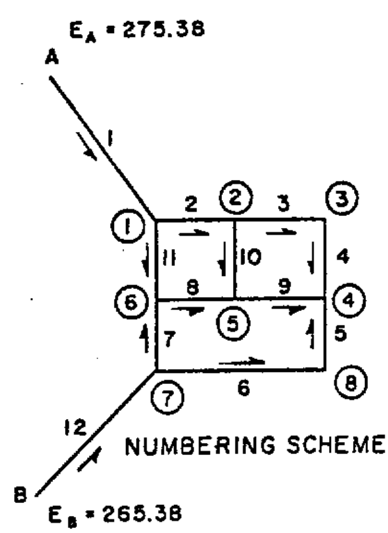
Example 5 - Linear Method

TRIAL NO.	RELATIVE FLOW CHANGE	AVERAGE FLOW CHANGE
1	.726257	1.18223
2	.111484	.181558
3	5.26916E-03	8.59965E-03
4	1.03049E-04	1.68175E-04

PIPE NUMBER	FLOWRATE
1	4.29337
2	3.07667
3	.892328
4	.107672
5	.631429
6	1.63143
7	1.0752
8	2.29189
9	.476242
10	.184349
11	1.21669
12	3.70662

NODE NUMBER	GRADE
1	239.25
2	234.053
3	233.265
4	233.327
5	233.996
6	237.385
7	238.776
8	234.763

```
2500 DATA 12,8,1
2510 DATA 1500,10,120,0,0
2520 DATA 400,10,120,0,0
2530 DATA 600,10,120,0,0
2540 DATA 800,8,120,0,0
2550 DATA 700,8,120,0,0
2560 DATA 1000,10,120,0,0
2570 DATA 750,10,120,0,0
2580 DATA 450,10,120,0,0
2590 DATA 550,8,120,0,0
2600 DATA 800,10,120,0,0
2610 DATA 800,10,120,0,0
2620 DATA 1450,10,120,0,0
2630 DATA 275.38
2640 DATA 3,0,-1,2,11,1,1
2650 DATA 3,2,-2,3,10,1,2
2660 DATA 2,1,-3,4,1,3
2670 DATA 3,1,-4,-5,-9,1,4
2680 DATA 3,2,-8,9,-10,1,-9
2685 DATA 3,0,-7,8,-11,1,-8
2690 DATA 3,1,6,7,-12,1,-7
2700 DATA 2,1,5,-6,1,6
2710 DATA 0,4,2,10,-8,-11
2720 DATA 0,4,3,4,-9,-10
2730 DATA 0,5,8,9,-5,-6,7
2740 DATA 10,4,1,11,-7,-12
3000 END
```



Example 6 - Linear Method

Appendix 4 Computer Program for Single Node Adjustment Method - Node Equations

The program listing (BASIC) which utilizes the procedure presented on pages 9 to 10 is shown on the next page. Additional pertinent notation along with the line number where it first appears is defined below.

140	N1 - number of pumps
180	U(N), V(N) - connecting nodes for pipe
195	O(N) - ΣM
210	F(N) - grade for pipe connecting fixed grade node
220	C(N) - K_p (Equ. 6)
260	N2 - number of line with pump
300	Y(N) - number of pipes connecting junction node N
300	E(N) - external flowrate, junction node N
330	W(N, M) - pipes connecting junction node N
420	K(N) - loss coefficient pipe N (Equ. 23)
440	H8 - summation of Grade Changes
450	Q8 - summation of flow changes
460	Q9 - summation of flows
600	Q0 - initial assumption - line with pump (Equ. 31)
610	B1 - denominator of Equ. 29
620	H4 - numerator of Equ. 29
720	A1 - ΣQ (numerator of Equ. 28)
780	B1 - denominator of Equ. 28

The data requirements are summarized below. Pipes are numbered from 1 to P and junction nodes from 1 to J. Fixed grade nodes are numbered zero. The required data is:

1 first line: no. of pipes, no. of junction nodes, number of pumps, flow conversion factor

2 next P lines (one for each pipe section): first node, second node, length (ft.), diameter (in.), roughness, Σ minor loss coefficients, value for fixed grade if this pipes connects a fixed grade node (otherwise omit this data).

3 next line: pipe number for line with pump, useful pump power, repeat this data for each pump (omit this line if system has no pumps)

4 next J lines (one for each junction node), number of pipes connecting this junction node, assumed grade for this node, external demand for this node, pipe numbers for pipes connecting this node.

The program listing and examples follow.

BASIC LISTING - SINGLE NODE ADJUSTMENT METHOD

```
100 DIM H(20),Z(20),B(20)
110 DIM U(20),V(20),D(20),C(20),O(20)
120 DIM K(20),E(20),Y(20),F(20)
130 DIM W(20,5),Q(20)
140 READ P,J,N1,C1
150 T6=50
160 X=1.852
170 FOR N=1 TO P
180 READ U(N),V(N),L1,D1,R1,M1
190 D(N)=D1/12
195 O(N)=M1
200 IF U(N)+V(N)<>ABS(U(N)-V(N)) GO TO 220
210 READ F(N)
220 C(N)=4.73*L1/(R1**X*D(N)**4.87)
230 NEXT N
240 IF N1=0 GO TO 290
250 FOR N=1 TO N1
260 READ N2,P1
270 Z(N2)=P1*550/62.4
280 NEXT N
290 FOR N=1 TO J
300 READ Y(N),H(N),E(N)
310 E(N)=E(N)/C1
320 FOR M=1 TO Y(N)
330 READ W(N,M)
340 NEXT M
350 NEXT N
360 T5=0
370 PRINT 'TRIAL NO.  RELATIVE FLOW CHANGE  AVERAGE HEAD CHANGE'
380 T5=T5+1
390 IF T5>T6 GO TO 970
400 FOR N=1 TO P
410 Q3=ABS(Q(N))
420 K(N)=C(N)+.02517*O(N)*Q3**.148/D(N)**4
430 NEXT N
440 H8=0
450 Q8=0
460 Q9=0
470 FOR C=1 TO J
480 A1=E(C)
490 B1=0
500 FOR M=1 TO Y(C)
510 N=W(C,M)
520 J3=U(N)+V(N)-C
530 H1=H(C)
540 IF J3=0 GO TO 570
550 H2=H(J3)
560 GO TO 580
570 H2=F(N)
580 H3=ABS(H1-H2)
590 IF Z(N)=0 GO TO 740
600 Q0=(Z(N)/K(N))**(1/(1+X))
610 B=1/(X*K(N)*Q0**(X-1)+Z(N)/Q0**2)
620 H4=H1-H2+Z(N)/Q0-K(N)*Q0**X
630 IF U(N)=C GO TO 650
640 H4=H2-H1+Z(N)/Q0-K(N)*Q0**X
650 Q1=H4*B
660 Q0=Q0+Q1
670 IF ABS(Q1)>.001 GO TO 610
680 B1=B1+B
```

```
690 Q(N)=Q0
700 IF U(N)=C GO TO 720
710 Q(N)=-Q0
720 A1=A1+Q(N)
730 GO TO 800
740 Q(N)=(H3/K(N))**(1/X)
750 IF H1>H2 GO TO 780
760 Q(N)=-Q(N)
770 IF H3=0 GO TO 790
780 B1=B1+(H3/K(N))**(1/X-1)/(X*(K(N)))
790 A1=A1+Q(N)
800 NEXT M
810 H(C)=H(C)-A1/B1
820 H8=H8+ABS(A1/B1)
830 NEXT C
840 FOR N=1 TO P
850 IF T5=1 GO TO 890
860 Q7= ABS(Q(N))
870 Q8=Q8+Q7
880 Q9=Q9+ABS(B(N)-Q7)
890 B(N)=ABS(Q(N))
900 NEXT N
910 IF T5=1 GO TO 960
920 Q5=Q9/Q8
930 H9=H8/J
940 PRINT T5,Q5,H9
950 IF Q5<.001 GO TO 970
960 GO TO 380
970 PRINT
980 PRINT
990 PRINT 'PIPE NUMBER      FLOWRATE'
1000 FOR N=1 TO P
1010 Q(N)=C1*ABS(Q(N))
1020 PRINT N,Q(N)
1030 NEXT N
1040 PRINT
1050 PRINT 'NODE NUMBER         GRADE'
1060 FOR N=1 TO J
1070 PRINT N,H(N)
1080 NEXT N
```

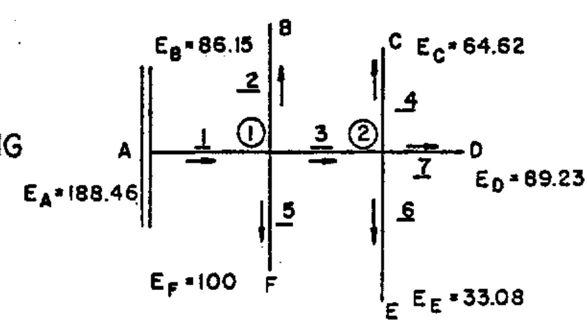
TRIAL NO.	RELATIVE FLOW CHANGE	AVERAGE HEAD CHANGE
2	.202666	2.16941
3	5.10954E-02	.730122
4	9.05142E-03	.119714
5	2.02572E-03	3.20134E-02
6	4.41792E-04	5.92414E-03

PIPE NUMBER	FLOWRATE
1	608.616
2	95.612
3	448.653
4	82.155
5	64.2859
6	80.5526
7	285.84

NODE NUMBER	GRADE
1	130.431
2	97.4404

```
1090 DATA 7,2,0,448.86
1100 DATA 0,1,200,4,110,0,188.46
1110 DATA 1,0,150,2,110,2,86.15
1120 DATA 1,2,200,4,110,0
1130 DATA 2,0,100,2,110,11,64.62
1140 DATA 1,0,200,2,110,6,100
1150 DATA 2,0,300,2,110,4,33.08
1160 DATA 2,0,80,4,110,3,89.23
1170 DATA 4,140,0,1,2,3,5
1180 DATA 4,100,0,3,4,6,7
```

NUMBERING SCHEME



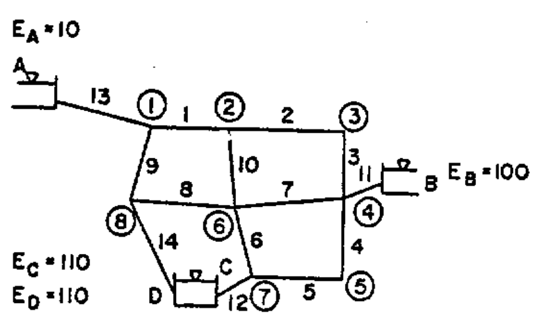
Example 1 - Single Node Adjustment

TRIAL NO.	RELATIVE FLOW CHANGE	AVERAGE HEAD CHANGE
2	.221078	3.74231
3	7.14057E-02	1.69595
4	2.81430E-02	.713241
5	1.63690E-02	.258435
6	7.16994E-03	.111082
7	2.83070E-03	7.31062E-02
8	1.26856E-03	4.41590E-02
9	5.68328E-04	2.54497E-02

PIPE NUMBER	FLOWRATE
1	9.65339
2	5.27404
3	1.27564
4	1.51875E-02
5	1.96641
6	3.36627
7	4.92659
8	3.90005
9	9.12071
10	4.38304
11	2.18904
12	.595121
13	18.7729
14	3.22902

NODE NUMBER	GRADE
1	202.413
2	133.421
3	104.607
4	102.871
5	102.869
6	119.764
7	106.473
8	136.2

```
1420 DATA 14,8,1,1
1430 DATA 1,2,2000,14,100,8
1440 DATA 2,3,3000,14,100,0
1450 DATA 3,4,2500,14,100,0
1460 DATA 4,5,2500,14,100,0
1470 DATA 5,7,2000,14,100,10
1480 DATA 6,7,1500,12,100,0
1490 DATA 4,6,2000,14,100,0
1500 DATA 6,8,3000,14,100,0
1510 DATA 1,8,2500,14,100,0
1520 DATA 2,6,2000,14,100,0
1530 DATA 4,0,100,8,100,0,100
1540 DATA 7,0,200,6,100,10,110
1550 DATA 0,1,5000,16,100,5,10
1560 DATA 8,0,3200,12,100,0,110
1570 DATA 13,1000
1580 DATA 3,160,0,1,9,13
1590 DATA 3,120,0,1,2,10
1600 DATA 2,100,4,2,3
1610 DATA 4,100,4,3,4,7,11
1620 DATA 2,100,2,4,5
1630 DATA 4,110,0,6,7,8,10
1640 DATA 3,105,2,5,6,12
1650 DATA 3,130,2,8,9,14
```



NUMBERING SCHEME

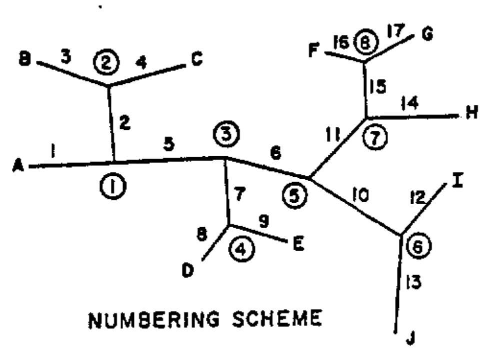
Example 2 - Single Node Adjustment

TRIAL NO.	RELATIVE FLOW CHANGE	AVERAGE HEAD CHANGE
2	.347826	7.62805
3	.112614	5.25576
4	.108164	2.10533
5	2.91173E-02	1.89873
6	1.62129E-02	1.36074
7	8.85858E-03	.988537
8	5.31508E-03	.789229
9	3.49220E-03	.662655
10	2.54442E-03	.57467
11	2.18855E-03	.508526
12	2.01639E-03	.455624
13	1.85296E-03	.411274
14	1.69353E-03	.372833
15	1.54191E-03	.338888
16	1.40171E-03	.308439
17	1.28043E-03	.280915
18	1.16707E-03	.256032
19	1.06388E-03	.233352
20	9.69746E-04	.212669

PIPE NUMBER	FLOWRATE
1	637.984
2	104.957
3	51.9648
4	52.9127
5	532.831
6	328.521
7	203.594
8	107.267
9	94.476
10	159.937
11	166.87
12	86.2587
13	71.3747
14	53.4727
15	109.372
16	48.9964
17	53.5407

NODE NUMBER	GRADE
1	288.727
2	154.929
3	186.595
4	171.828
5	157.974
6	148.014
7	151.74
8	149.717

```
2000 DATA 17,8,1,448.86
2010 DATA 0,1,635,4,130,0,100
2020 DATA 1,2,520,2,130,5
2030 DATA 2,0,380,2,130,23,120
2040 DATA 2,0,510,2,130,23,110
2050 DATA 1,3,615,4,130,0
2060 DATA 3,5,425,4,130,0
2070 DATA 3,4,385,4,130,10
2080 DATA 4,0,195,2,130,23,80
2090 DATA 4,0,270,2,130,23,85
2100 DATA 5,6,540,4,130,2
2110 DATA 5,7,335,4,130,0
2120 DATA 6,0,210,2,130,23,85
2130 DATA 6,0,415,2,130,23,80
2140 DATA 7,0,375,2,130,23,115
2150 DATA 7,8,255,4,130,0
2160 DATA 8,0,180,2,130,23,130
2170 DATA 8,0,200,2,130,23,125
2190 DATA 1,54.2
2200 DATA 3,300,0,1,2,5
2210 DATA 3,150,0,2,3,4
2220 DATA 3,200,0,5,6,7
2230 DATA 3,150,0,7,8,9
2240 DATA 3,150,0,6,10,11
2250 DATA 3,100,0,10,12,13
2260 DATA 3,125,0,11,14,15
2270 DATA 3,100,0,15,16,17
```



NUMBERING SCHEME

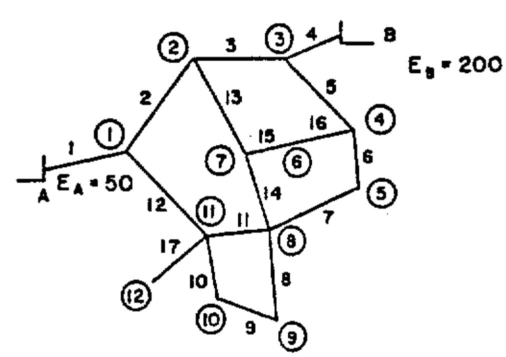
Example 4 - Single Node Adjustment

TRIAL NO.	RELATIVE FLOW CHANGE	AVERAGE HEAD CHANGE
2	.318495	12.2624
3	.159235	6.08033
4	7.65380E-02	3.41788
5	3.18711E-02	1.03428
6	1.59870E-02	.572319
7	6.45460E-03	.31493
8	2.64258E-03	.238975
9	1.17918E-03	.217959
10	8.77676E-04	.212677

PIPE NUMBER	FLOWRATE
1	29.2392
2	16.3758
3	6.13574
4	2.71625
5	3.4358
6	.565217
7	4.51674
8	1.3771
9	.591566
10	2.58665
11	8.50433
12	12.8706
13	5.25469
14	2.64324
15	2.94208
16	.107658
17	1.87483

NODE NUMBER	GRADE
1	485.115
2	388.502
3	360.587
4	340.449
5	341.251
6	340.355
7	348.378
8	374.204
9	367.555
10	388.903
11	404.545
12	401.766

```
2500 DATA 17,12,1,1
2510 DATA 0,1,10000,24,110,10,50
2520 DATA 1,2,5000,18,110,0
2530 DATA 2,3,5000,16,110,0
2540 DATA 3,0,1000,6,110,5,200
2550 DATA 3,4,5500,14,110,0
2560 DATA 4,5,3500,12,110,0
2570 DATA 8,5,5500,14,110,0
2575 DATA 8,9,4500,12,110,0
2580 DATA 10,9,2500,6,110,0
2590 DATA 11,10,3500,12,110,0
2600 DATA 11,8,2200,15,110,0
2610 DATA 1,11,6500,18,110,0
2620 DATA 2,7,5000,14,110,0
2630 DATA 8,7,5500,12,110,0
2640 DATA 7,6,3000,14,110,0
2650 DATA 6,4,4000,12,110,0
2660 DATA 11,12,4000,16,110,0
2680 DATA 1,1950
2690 DATA 3,500,0,1,2,12
2700 DATA 3,400,5,2,3,13
2710 DATA 3,350,0,3,4,5
2720 DATA 3,300,4,5,6,16
2730 DATA 2,300,4,6,7
2740 DATA 2,350,3,15,16
2750 DATA 3,375,5,13,14,15
2760 DATA 4,350,0,7,8,11,14
2770 DATA 2,300,2,8,9
2780 DATA 2,350,2,9,10
2790 DATA 4,400,0,10,11,12,17
2800 DATA 1,350,2,17
```



NUMBERING SCHEME

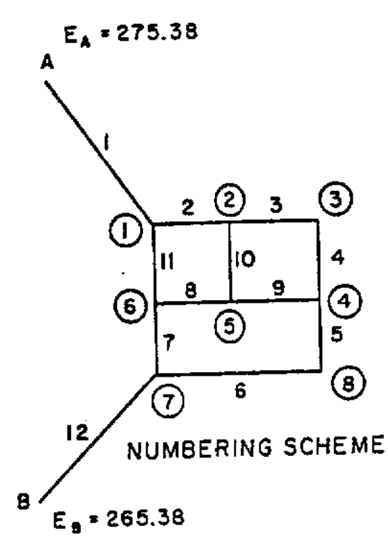
Example 5 - Single Node Adjustment

TRIAL NO.	RELATIVE FLOW CHANGE	AVERAGE HEAD CHANGE
2	.235817	2.63599
3	.287243	1.03603
4	.109522	.250238
5	4.60690E-02	6.67058E-02
6	1.82453E-02	3.40214E-02
7	6.27008E-03	1.17870E-02
8	2.06382E-03	1.06792E-02
9	8.35856E-04	1.09384E-02

PIPE NUMBER	FLOWRATE
1	4.32484
2	3.10138
3	.905732
4	.102327
5	.640531
6	1.64572
7	1.09048
8	2.30686
9	.474756
10	.190984
11	1.223
12	3.74085

NODE NUMBER	GRADE
1	238.765
2	233.501
3	232.706
4	232.776
5	233.452
6	236.892
7	238.329
8	234.261

```
1200 DATA 12,8,0,1
1210 DATA 0,1,1500,10,120,0,275.38
1220 DATA 1,2,400,10,120,0
1230 DATA 2,3,600,10,120,0
1240 DATA 3,4,800,8,120,0
1250 DATA 8,4,700,8,120,0
1260 DATA 7,8,1000,10,120,0
1270 DATA 7,6,750,10,120,0
1280 DATA 6,5,450,10,120,0
1290 DATA 5,4,550,8,120,0
1300 DATA 2,5,800,10,120,0
1310 DATA 1,6,800,10,120,0
1320 DATA 0,7,1450,10,120,0,265.38
1330 DATA 3,240,0,1,2,11
1340 DATA 3,230,2,2,3,10
1350 DATA 2,238,1,3,4
1360 DATA 3,237,1,4,5,9
1370 DATA 3,236,2,8,9,10
1380 DATA 3,235,0,7,8,11
1390 DATA 3,234,1,6,7,12
1400 DATA 2,233,1,5,6
```



Example 6 - Single Node Adjustment

Appendix 5 Computer Program for Simultaneous Node Adjustment Method - Node Equations

The programs listing (BASIC) which utilizes the procedure presented on pages 10 to 12 in shown on the next page. To help clarify the program Equ. 35 is written as

$$A_1H_b - A_5H_a = Q_e \pm B_2 \pm B_4 - B_3$$

where A_1, A_5, B_2, B_3 and B_4 are program variables which represent the respective summations shown in Equ. 35. The program notation for this program is very similar to notation presented in Appendix 4 for the single node adjustment method. The notations defined in lines 140 through 460 in Appendix 3 are identical and will not be repeated here. Additional pertinent notation along with the line number where it first appears.

310	T(N), T(N+N) - pipe numbers for suction and discharge lines for pump number N
490	H1 - grade at one end of pipe section
530	H2 - grade at other end of pipe section
590	S(N) - α for pipe N (Equ. 34).
860	A(N,J3) - linearized equation coefficient for H_b
970	A(N,N) - linearized equation coefficient for H_a
980	B(N,1) - constant (right side) for linearized equation
1010	L1 - suction line number for pump
1020	L2 - discharge line number for pump
1030	L3 - upstream node number for suction line
1040	L4 - downstream node number for suction line
1050	L5 - upstream node number for discharge line
1060	L6 - downstream node number for discharge line
1160	B1 - B (Equ. 37)
1170	B(N3,1) - W(Equ. 38)

For data input the pipes are numbered 1 to P and junction nodes 1 to J noting that a pump requires a separate suction and discharge line and junction nodes must be identified at the suction and discharge sides of the pump. When this is done the data coding is carried out using the same instructions as given in Appendix 4 for the single node adjustment method with one exception which is noted for item 3:

3 pipe number for suction line, pipe number for discharge line, useful pump power - repeat this data for each pump (omit this line if system has no pumps).

A comparison of Examples 2, 4, and 5 for the single and simultaneous node adjustment methods should clarify the data input procedures.

The program listing and examples follow.

BASIC LISTING - SIMULTANEOUS NODE ADJUSTMENT METHOD

```
100 DIM A(20,20),R(20,20),B(20,1),G(20,1)
110 DIM U(20),V(20),D(20),C(20),O(20)
120 DIM K(20),E(20),Y(20),F(20),S(20)
130 DIM W(20,5),Q(20),H(20)
140 DIM I(20),T(20),Z(20)
150 READ P,J,N1,C1
160 MAT A = ZER(J,J)
170 MAT R = ZER(J,J)
180 MAT B = ZER(J,1)
190 MAT G = ZER(J,1)
200 X=1.852
210 FOR N=1 TO P
220 READ U(N),V(N),L1,D1,R1,M1
230 D(N)=D1/12
235 O(N)=M1
240 IF U(N)+V(N)<>ABS(U(N)-V(N)) GO TO 260
250 READ F(N)
260 C(N)=4.73*L1/(R1**X*D(N)**4.87)
270 K(N)=C(N)
280 NEXT N
290 IF N1=0 GO TO 340
300 FOR N=1 TO N1
310 READ T(N),T(N+N1),P1
320 Z(N)=P1*550/62.4
330 NEXT N
340 J1=J-2*N1
350 FOR N=1 TO J1
360 READ Y(N),H(N),E(N)
370 E(N)=E(N)/C1
380 FOR M=1 TO Y(N)
390 READ W(N,M)
400 NEXT M
410 NEXT N
420 PRINT 'TRIAL NO.  RELATIVE FLOW CHANGE  AVERAGE HEAD CHANGE'
430 FOR T5=1 TO 40
440 Q8=0
450 Q9=0
460 H8=0
470 FOR N=1 TO P
480 IF U(N)=0 GO TO 510
490 H1=H(U(N))
500 GO TO 520
510 H1=F(N)
520 IF V(N)=0 GO TO 550
530 H2=H(V(N))
540 GO TO 560
550 H2=F(N)
560 Q3=(ABS(H1-H2)/K(N))**(1/X)
570 Q(N)=Q3
580 K(N)=C(N)+.02517*O(N)*Q3**.148/D(N)**4
590 S(N)=ABS((H1-H2)/K(N))
600 IF T5=1 GO TO 640
610 Q8=Q8+Q3
620 Q9=Q9+ABS(I(N)-Q3)
630 I(N)=Q3
640 NEXT N
650 IF T5=1 GO TO 690
660 Q5=Q9/Q8
670 PRINT T5,Q5,H9
680 IF Q5<.001 GO TO 1300
690 FOR N=1 TO J
```

```
700 FOR M=1 TO J
710 A(N,M)=0
720 NEXT M
730 NEXT N
740 FOR N=1 TO J1
750 A1=0
760 B1=0
770 FOR M=1 TO Y(N)
780 L=W(N,M)
790 J3=U(L)+V(L)-N
800 IF S(L)=0. GO TO 960
810 A5=S(L)**(1/X-1)/(K(L)*X)
820 A1=A1+A5
830 B2=-(S(L)**(1/X))
840 B4=-B2/X
850 IF J3=0 GO TO 920
860 A(N,J3)=A(N,J3)+A5
870 IF H(J3)>H(N) GO TO 900
880 B2=-B2
890 B4=-B4
900 B1=B1+B2+B4
910 GO TO 960
920 B3=-H(N)*A5
930 IF F(L)>H(N) GO TO 950
940 B2=-B2
950 B1=B1+B2+B3
960 NEXT M
970 A(N,N)=-A1
980 B(N,1)=B1+E(N)
990 NEXT N
1000 FOR N=1 TO N1
1010 L1=T(N)
1020 L2=T(N+N1)
1030 L3=U(L1)
1040 L4=V(L1)
1050 L5=U(L2)
1060 L6=V(L2)
1070 N2=J1+2*N-1
1080 N3=N2+1
1090 IF L3<>0 GO TO 1120
1100 B(N2,1)=-F(L1)
1110 GO TO 1130
1120 A(N2,L3)=1
1130 A(N2,L4)=-1
1140 A(N2,L5)=-K(L1)/K(L2)
1150 A(N2,L6)=K(L1)/K(L2)
1160 B1=Z(N)/(X*K(L2)*S(L2)**(1/X+1))
1170 B(N3,1)=Z(N)*(1+1/X)/S(L2)**(1/X)
1180 A(N3,L4)=-1
1190 A(N3,L5)=1+B1
1200 A(N3,L6)=-B1
1210 NEXT N
1220 MAT R=INV(A)
1230 MAT G =R*B
1240 FOR N=1 TO J
1250 H8=H8+ABS(H(N)-G(N,1))
1260 H(N)=G(N,1)
1270 NEXT N
1280 H9=H8/J
1290 NEXT T5
1300 PRINT
1310 PRINT
1320 PRINT 'PIPE NUMBER        FLOWRATE'
1330 FOR N=1 TO P
1340 Q(N)=C1*ABS(Q(N))
1350 PRINT N,Q(N)
1360 NEXT N
1370 PRINT
1380 PRINT 'NODE NUMBER         GRADE'
1390 FOR N=1 TO J
1400 PRINT N,H(N)
1410 NEXT N
```

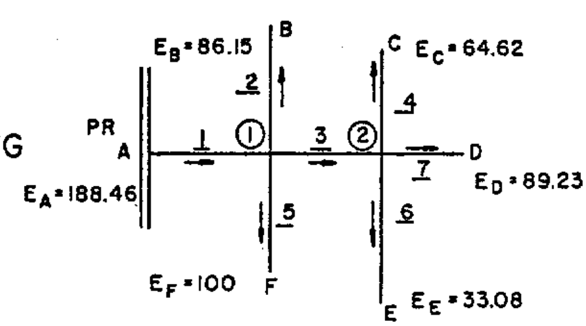
```
TRIAL NO.  RELATIVE FLOW CHANGE  AVERAGE HEAD CHANGE
 2                 1                 6.00331
 3                 1.20125E-03       6.05087E-02
 4                 2.41563E-05       1.06812E-03

PIPE NUMBER        FLOWRATE
 1                  608.558
 2                  95.6236
 3                  448.636
 4                  82.16
 5                  64.2971
 6                  80.5557
 7                  285.918

NODE NUMBER        GRADE
 1                  130.433
 2                  97.4412
```

```
1420 DATA 7,2,0,448.86
1430 DATA 0,1,200,4,110,0,188.46
1440 DATA 1,0,150,2,110,2,86.15
1450 DATA 1,2,200,4,110,0
1460 DATA 2,0,100,2,110,11,64.62
1470 DATA 1,0,200,2,110,6,100
1480 DATA 2,0,300,2,110,4,33.08
1490 DATA 2,0,80,4,110,3,89.23
1520 DATA 4,140,0,1,2,3,5
1530 DATA 4,100,0,3,4,6,7
```

NUMBERING SCHEME



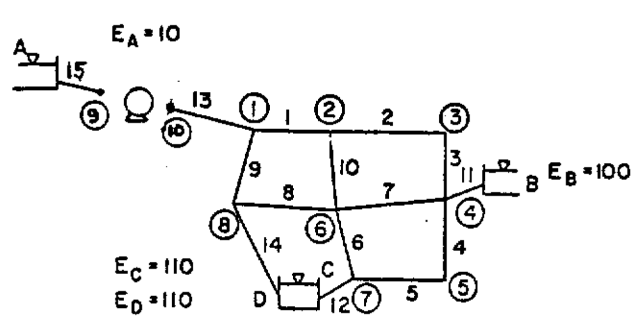
Example 1 - Simultaneous Node Adjustment

TRIAL NO.	RELATIVE FLOW CHANGE	AVERAGE HEAD CHANGE
2	1	87.0873
3	.203535	118.659
4	.307593	73.6377
5	.184764	31.8814
6	.159278	11.9253
7	6.16838E-02	8.85837
8	5.08917E-02	3.38077
9	1.77257E-02	.640245
10	1.13498E-02	.422391
11	7.54545E-03	.292046
12	6.25869E-03	.21102
13	3.92097E-03	.154053
14	3.77411E-03	.116092
15	2.13284E-03	8.72378E-02
16	2.47577E-03	6.75436E-02
17	1.16367E-03	5.15275E-02
18	1.73981E-03	4.06568E-02
19	7.39667E-04	3.14178E-02

PIPE NUMBER	FLOWRATE
1	9.65661
2	5.27136
3	1.27136
4	.249994
5	1.97422
6	3.36835
7	4.91548
8	3.89862
9	9.11989
10	4.38526
11	2.16105
12	.605895
13	18.7765
14	3.22128
15	18.7765

NODE NUMBER	GRADE
1	202.286
2	133.299
3	104.528
4	102.807
5	102.722
6	119.659
7	106.362
8	136.114
9	-.52384
10	468.897

```
1800 DATA 15,10,1,1
1810 DATA 1,2,2000,14,100,8
1820 DATA 2,3,3000,14,100,0
1850 DATA 3,4,2500,14,100,0
1860 DATA 4,5,2500,14,100,0
1870 DATA 5,7,2000,14,100,10
1880 DATA 6,7,1500,12,100,0
1890 DATA 4,6,2000,14,100,0
1900 DATA 6,8,3000,14,100,0
1910 DATA 1,8,2500,14,100,0
1920 DATA 2,6,2000,14,100,0
1930 DATA 4,0,100,8,100,0,100
1940 DATA 7,0,200,6,100,10,110
1950 DATA 10,1,4800,16,100,5
1960 DATA 8,0,3200,12,100,0,110
1965 DATA 0,9,200,16,100,0,10
1970 DATA 15,13,1000
1980 DATA 3,160,0,1,9,13
1990 DATA 3,120,0,1,2,10
2000 DATA 2,100,4,2,3
2010 DATA 4,100,4,3,4,7,11
2020 DATA 2,100,2,4,5
2030 DATA 4,110,0,6,7,8,10
2040 DATA 3,105,2,5,6,12
2050 DATA 3,130,2,8,9,14
```



NUMBERING SCHEME

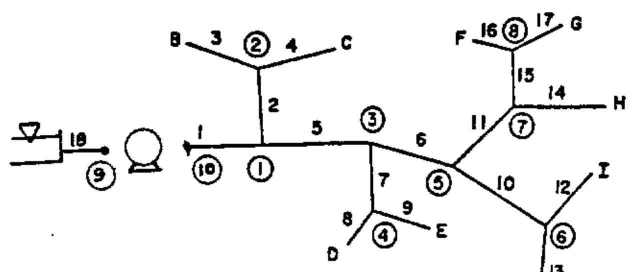
Example 2 - Simultaneous Node Adjustment

TRIAL NO.	RELATIVE FLOW CHANGE	AVERAGE HEAD CHANGE
2	1	108.884
3	.775802	130.019
4	.41116	153.76
5	.281012	127.139
6	.796696	49.2704
7	.316552	40.2987
8	.206562	10.051
9	2.86635E-02	1.73391
10	3.23405E-03	.107019
11	1.25353E-04	3.92456E-03

PIPE NUMBER	FLOWRATE
1	635.786
2	105.558
3	52.3437
4	53.2143
5	530.23
6	325.627
7	204.602
8	108.743
9	95.8605
10	160.979
11	164.647
12	88.1411
13	72.8387
14	55.7038
15	108.944
16	52.4118
17	56.5353
18	635.785

NODE NUMBER	GRADE
1	290.641
2	155.371
3	189.182
4	174.058
5	160.76
6	150.44
7	154.425
8	152.18
9	76.9093
10	414.177

```
2100 DATA 18,10,1,448.86
2110 DATA 10,1,535,4,130,0
2120 DATA 1,2,520,2,130,5
2130 DATA 2,0,380,2,130,23,120
2140 DATA 2,0,510,2,130,23,110
2150 DATA 1,3,615,4,130,0
2160 DATA 3,5,425,4,130,0
2170 DATA 3,4,385,4,130,10
2180 DATA 4,0,195,2,130,23,80
2190 DATA 4,0,270,2,130,23,85
2200 DATA 5,6,540,4,130,2
2210 DATA 5,7,335,4,130,0
2220 DATA 6,0,210,2,130,23,85
2230 DATA 6,0,415,2,130,23,80
2240 DATA 7,0,375,2,130,23,115
2250 DATA 7,8,255,4,130,0
2260 DATA 8,0,180,2,130,23,130
2270 DATA 8,0,200,2,130,23,125
2275 DATA 0,9,100,4,130,0,100
2290 DATA 18,1,54.2
2300 DATA 3,300,0,1,2,5
2310 DATA 3,150,0,2,3,4
2320 DATA 3,200,0,5,6,7
2330 DATA 3,150,0,7,8,9
2340 DATA 3,150,0,6,10,11
2350 DATA 3,100,0,10,12,13
2360 DATA 3,125,0,11,14,15
2370 DATA 3,100,0,15,16,17
```



NUMBERING SCHEME

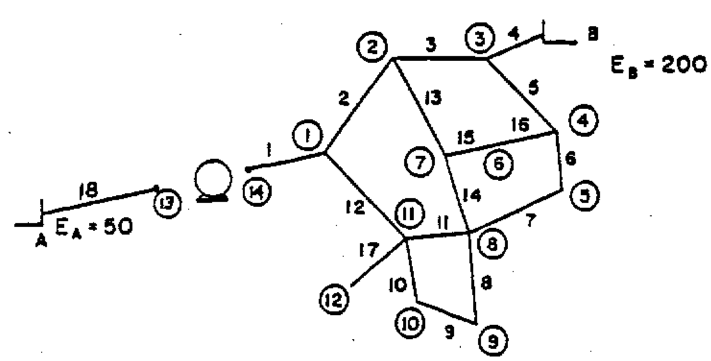
Example 4 - Simultaneous Node Adjustment

TRIAL NO.	RELATIVE FLOW CHANGE	AVERAGE HEAD CHANGE
2	1	820.822
3	.736709	933.51
4	.289404	688.393
5	.556325	629.608
6	.267509	169.345
7	.108602	110.453
8	7.04484E-02	14.0286
9	3.27783E-02	4.76694
10	2.75472E-02	3.63112
11	1.59273E-02	2.46052
12	1.61131E-02	1.85413
13	1.01767E-02	1.22515
14	1.17714E-02	.884266
15	7.17961E-03	.466602
16	3.90817E-03	.320872
17	3.39971E-03	.232746
18	1.70053E-03	.168585
19	2.11461E-03	.126763
20	1.03697E-03	9.20595E-02
21	1.46189E-03	7.10275E-02
22	7.26101E-04	5.16030E-02

PIPE NUMBER	FLOWRATE
1	29.6046
2	16.5054
3	6.15971
4	2.60488
5	3.55491
6	.488296
7	4.48843
8	1.41212
9	.587806
10	2.58768
11	8.51142
12	13.0994
13	5.34551
14	2.6109
15	2.95645
16	.229796
17	1.99969
18	29.6045

NODE NUMBER	GRADE
1	474.498
2	376.627
3	348.638
4	327.324
5	328.052
6	327.118
7	335.382
8	360.877
9	354.194
10	375.612
11	391.57
12	388.783
13	48.5772
14	629.142

```
2500 DATA 18,14,1,1
2510 DATA 14,1,9900,24,110,10
2520 DATA 1,2,5000,18,110,0
2530 DATA 2,3,5000,16,110,0
2540 DATA 3,0,1000,6,110,5,200
2550 DATA 3,4,5500,14,110,0
2560 DATA 4,5,3500,12,110,0
2570 DATA 8,5,5500,14,110,0
2575 DATA 8,9,4500,12,110,0
2580 DATA 10,9,2500,6,110,0
2590 DATA 11,10,3500,12,110,0
2600 DATA 11,8,2200,15,110,0
2610 DATA 1,11,6500,18,110,0
2620 DATA 2,7,5000,14,110,0
2630 DATA 8,7,5500,12,110,0
2640 DATA 7,6,3000,14,110,0
2650 DATA 6,4,4000,12,110,0
2670 DATA 0,13,100,24,110,0,50
2680 DATA 18,1,1950
2690 DATA 3,500,0,1,2,12
2700 DATA 3,400,5,2,3,13
2710 DATA 3,350,0,3,4,5
2720 DATA 3,300,4,5,6,16
2730 DATA 2,300,4,6,7
2740 DATA 2,350,3,15,16
2750 DATA 3,375,5,13,14,15
2760 DATA 4,350,0,7,8,11,14
2770 DATA 2,300,2,8,9
2780 DATA 2,350,2,9,10
2790 DATA 4,400,0,10,11,12,17
2800 DATA 1,350,2,17
2810 END
```



NUMBERING SCHEME

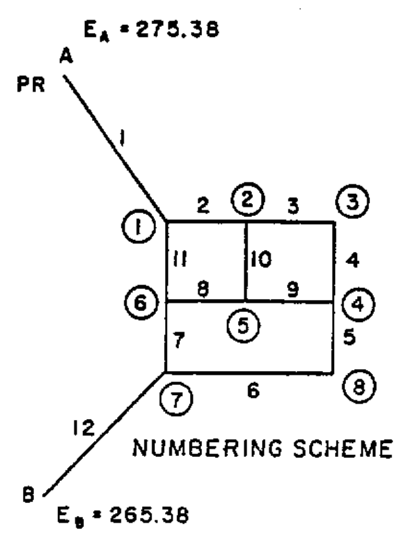
Example 5 - Simultaneous Node Adjustment

TRIAL NO.	RELATIVE FLOW CHANGE	AVERAGE HEAD CHANGE
2	1	5.24186
3	.374602	3.65636
4	.149486	2.27042
5	.178418	1.41315
6	7.02658E-02	.870077
7	.148887	.554546
8	6.97266E-02	.160944
9	3.56591E-02	.121954
10	1.20965E-02	4.80423E-02
11	5.72392E-04	2.40898E-03

PIPE NUMBER	FLOWRATE
1	4.2934
2	3.07665
3	.892172
4	.107563
5	.631397
6	1.63141
7	1.07516
8	2.29186
9	.476224
10	.18419
11	1.21667
12	3.70666

NODE NUMBER	GRADE
1	239.25
2	234.052
3	233.265
4	233.327
5	233.996
6	237.385
7	238.775
8	234.762

```
1540 DATA 12,8,0,1
1550 DATA 0,1,1500,10,120,0,275.38
1560 DATA 1,2,400,10,120,0
1570 DATA 2,3,600,10,120,0
1580 DATA 3,4,800,8,120,0
1590 DATA 8,4,700,8,120,0
1600 DATA 7,8,1000,10,120,0
1610 DATA 7,6,750,10,120,0
1620 DATA 6,5,450,10,120,0
1630 DATA 5,4,550,8,120,0
1640 DATA 2,5,800,10,120,0
1650 DATA 1,6,800,10,120,0
1660 DATA 0,7,1450,10,120,0,265.38
1670 DATA 3,240,0,1,2,11
1680 DATA 3,230,2,2,3,10
1690 DATA 2,238,1,3,4
1700 DATA 3,237,1,4,5,9
1710 DATA 3,236,2,8,9,10
1720 DATA 3,235,0,7,8,11
1730 DATA 3,234,1,6,7,12
1740 DATA 2,233,1,5,6
```



Example 6 - Simultaneous Node Adjustment